Contemporary Mathematics

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Teachers' Guide

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Contemporary Mathematics

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Contemporary Mathematics 1 is the seventh book in the Holt, Rinehart and Winston Canadian arithmetic programme.

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INTRODUCTION

It has been said that the modern revolution in arithmetic is as much one of viewpoint as of content. Certainly the new view of the traditional content of elementary school arithmetic is as important as any new content *per se*. This traditional content is still the mainstream of elementary mathematics, and failure to use it in developing the new ideas in the programme is more serious than the omission of the peculiarly modern topics.

At one time, industry, commerce and science demanded of the school-leaver a high standard of purely computational skill, and the arithmetic programme was designed to produce human calculating machines. *Insight* was developed either from perceptive high-school teaching or later from the university.

But during the last thirty years first electric and, more recently, electronic calculating machines have obviated the need for such emphasis on high-speed computation. At the same time the demand for people who have mathematical *understanding* has expanded exponentially. It is difficult to predict the technical skills that our students will need to acquire within the short period that lies between the seventh and twelfth grades. But the technical skills of the future will probably be based on the same mathematical ideas as those of the present.

Secondary programmes are now in a state of flux in an attempt to match programmes with the needs of students, and they are likely to remain so in the foreseeable future. The mathematical bases introduced at the elementary level are fundamentally the same no matter how they are extended and applied at the secondary level.

The contrasts between the traditional and the new approaches are summarized as follows:

a Instead of being considered as independent rituals, the algorithms of arithmetic are now studied as applications of a few underlying principles. Through the early grades and, much more explicitly, in the seventh and eighth grades, these principles are identified and consciously used in arithmetical discussion.

Towards the ninth grade these principles become increasingly associated with open sentences. They control variables rather than describe arithmetic. Thus, the student has begun algebra, but what has been called the "shut up—do this" approach is replaced by the conscious exercise of a small group of principles. Most teachers who have taught both the traditional and the modern programme agree that students who learn in this second way come to new material with a confidence and mastery previously unknown.

b The grouping of problems according to *topic* which, at the worst led to the mere writing of statements, is now replaced by a grouping according to mathematical structure.

These structures, or relations between numbers, are the same patterns that students have always found in problems: it is not surprising then, that students master them better when they are taught to use them as a starting point. These patterns are first encountered in open sentences; later, simple

> verbiage clothes the structure; then it is found in more complicated situations. Eventually, in science and technology, this task of identifying structure becomes the most important one.

Arithmetic and problem-solving in this wider context demand not only a different view of traditional topics, but also revisions in sequence of presentation. The teacher who is new to this approach would do well to study the sequence of topics carefully before beginning the year's work.

In the initial stages, it may be found that the more thoroughly a class has been drilled in the traditional methods, the more difficult will be its introduction to the new approach. Even bright students may feel that their time is being wasted when they are asked to "think about" a process. Experience suggests that this phase may take about two months, after which the joy of achievement overcomes the uncertainty of the new. Many teachers who have experienced such an awakening in a class, regardless of its level of ability, report results far beyond their expectations.

Accuracy and understanding are the new virtues, and speed for its own sake is avoided. For this reason the more traditional standardized speed tests are inappropriate to the assessment of progress in a course of this nature. Until more appropriate tests are constructed the teacher will have to depend on his own resources. The chapter tests in the text will be useful in suggesting the kinds of question that might be given.

Above all, the teacher should become sufficiently familiar with the new programme of school mathematics to be able to judge for himself whether test items are suitable or whether they might prejudice his teaching.

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1

The Whole Numbers

There are many situations in school mathematics in which the language of sets facilitates economy of expression. Since the language rather than the theory of sets is the more useful until about the twelfth grade, the degree to which it is introduced in a particular grade must be related to its utility in that grade.

In **Contemporary Mathematics I** it has been found sufficient to use the idea of a set and its elements, matching and identical sets, finite and infinite sets, and the empty set.

The concepts of *intersection* and *union* are mentioned where appropriate in the *Teachers' Guide*, but their introduction in a text is postponed until **Contemporary Mathematics II**.

WHAT IS A SET? (Page 1)

The first paragraph of this section makes it clear that a set may have elements of any sort; it is not necessary that the elements of a set be *numbers*. Later in the year the notion of a set of lines or a set of points will also be useful. All of the sets mentioned in this text will have some obvious family relationship; they are all numbers, or all points, or all lines. The teacher should realize that there is no such restriction upon membership in a set in general. A set may have as elements the number three, the Canadian flag, the Pacific Ocean, and the idea of justice. This is not a very likely set, but it is legitimately a set.

It is a good plan to consider here sets which contain only one element.

NATURAL NUMBERS AND THE WHOLE NUMBERS (Pages 1 and 2)

There are two ways of comparing different but similar games. If we were looking at Canadian and American football we could say, "Start with Canadian football. Remove one player from each team. Reduce the downs from four to three . . . ." When we were finished listing the modifications we would have given a fair description of American football. The other way would be to describe both games fully and then note the similarities between the games.

Mathematicians have always preferred the second approach when they are describing the systems of mathematics. Generally, mathematical systems are not described as modifications of other systems. At the same time, the mathematician and the student might well *think* of a new system as a modification of an old one.

Two different but very closely related systems are the natural numbers and the whole numbers. The natural numbers are the numbers

$$1, 2, 3, 4, \ldots$$

The whole numbers are the numbers

$$0, 1, 2, 3, \ldots$$

If we were to be quite fussy mathematically we would use a different set of symbols in the second list from that in the first list. We are working in different systems and we would write something like

$$\begin{array}{cccccccl} & 1 & 2 & 3 & 4 & 5 & \quad & \text{The Natural Numbers} \\ \bar{0} & \bar{1} & \bar{2} & \bar{3} & \bar{4} & \bar{5} & \quad & \text{The Whole Numbers} \end{array}$$

and then notice that the natural number 1 behaves exactly the same way as the whole number 1, that the natural number 2 behaves exactly the same way as the whole number 2, and so on through the two systems. This may seem an artificial way of looking at this situation, but we shall see when we study the fractional numbers that it is a most useful one.

At this stage it is not wise, except with the most able classes, to make this distinction. The section suggests that the whole numbers and the natural number are two different systems, but the transition from one to the other is thought of as being the inclusion of zero.

The symbol . . . as used in this section carries two burdens, but they should not be discussed formally with most classes. In an expression such as

$$N = \{1, 2, 3, 4, \ldots\}$$

the three dots first of all mean that we are to continue as we have been doing. Since no last numeral is given, the three dots also assert that we could continue to give numbers in this sequence as long as we like; there is no last natural number.

TWO USEFUL SIGNS (Pages 2 and 3)

Students in the seventh grade have worked with the ideas *greater than* and *less than* for several years. The signs > and < should be regarded as a convenient shorthand. It is a good plan to practise reading each sign both ways. For example, $9 > 3$ may be read as, "Nine is greater than three" or as "Three is less than nine". It is also a good plan to suggest the more formal way of regarding the ideas of *less than* and *greater than*. While the idea of order in the natural and whole numbers is clear to all students by the seventh grade, these ideas will be taken over to a study of the integers in the eighth grade where the notion is not so intuitively obvious. In more formal mathematics, the statement that $9 > 3$ means that some number must be added to three to get nine. (When the integers are studied, it will be asserted that a is greater than b if we must add some positive number to b to get a.)

IDENTICAL SETS (Pages 3 and 4)

The import of this section is that the elements of a set may be listed in any order.

The teacher should take care that a confusion between identical sets and matching sets is avoided. The sets {dog, cat, horse} and {1, 4, 9} are said to be *matching* sets because the elements of the two sets could be matched, one to one. They are certainly not *identical* sets.

The equals sign, =, is now being used with much more care in all school mathematics than was the case some years ago. This sign may only be used when the labels to the left and right of it name identically the same thing. This is why in geometry, for example, we no longer say that a pair of sides of different triangles are *equal*. Here, the equals sign is used quite correctly. In X = {2, 4, 6} and Y = {4, 6, 2}, the sign asserts that 'X' and {2,4,6} are different names for the same set. X = Y then means that 'X' and 'Y' name the same sets.

It is not necessary to undertake this formal discussion of the equals sign with most classes. It *is* essential that the teacher use the equals sign in this way and help each student develop this sense of the use of the sign.

FINITE AND INFINITE SETS (Page 4)

This section is intended to make more precise an idea first met in an earlier section. Nothing beyond an intuitive sense for the distinction between finite and infinite sets need be attempted. The student should sense that in a finite set, if he were to "look at" the elements of the set one after another he would eventually come to a last element. Similarly, any attempt to look at all the elements of an infinite set one after another must end in failure. There can be no last element.

The references to *counting* the elements of sets in the text are intended to convey this intuitive notion. The teacher should take care, however, to refer to sets as being *finite* rather than *countable*. In mathematics, the notion of *countable sets* means something quite different from *finite sets*. The distinction need not concern us here, but *students should not learn incorrect terminology, particularly when they are learning it for the first time.*

More able classes might discuss the idea of an infinite set at greater length. The teacher might ask whether or not the set of all the grains of sand on all the beaches in the world is a finite or infinite set. Once it is decided that this set is finite, the teacher might ask if the class can provide any example of an infinite set. The consequence of this discussion might be that the idea of an infinite set is completely a mathematical idea; there are no infinite sets in the real world.

THE EMPTY SET (Page 5)

The concept of the empty set is quite difficult, and there are problems about it that are well avoided at this level. The intent of this section is merely that students become aware of the fact that it is correct to think of a set with no elements. A good number of examples from everyday speech might be used. After discussing sets of objects in the classroom which are not empty, the teacher might refer to the set of all students over seven feet tall. The class might then even pass a regulation concerning such students (such as their being required to open the tops of windows). It is possible to discuss an empty set.

The teacher should make some attempt to differentiate between { } and {0}. The set {0} matches the sets {1}, {2}, {3} etc. They all have one element. The fact that the one element in {0} happens to be the number zero does not affect the number of elements in the set. The set { } is *empty*; it has no elements. In other words, the number of elements in {0} is *one*, the number of elements in { } is *zero*.

The teacher should realize, but should not attempt to teach, except perhaps with the most able classes, that there is only one empty set. It was pointed out earlier that a set A and a set B are identical if they have the same elements. Certainly two sets that are empty have identically the same elements, and are therefore the same set. This point is of some importance in the eighth grade, where the set of subsets of a set is considered, but may well be passed over here.

NUMBER AND NUMERAL

NUMERALS THAT WE USE

SOME WAYS OF WRITING NUMERALS (Pages 5 to 8)

Students who have worked with **Patterns in Arithmetic** in previous years have studied the distinction between number and numeral in some detail, and need only review the idea briefly here. Students who have not encountered this idea before should study it in considerable detail as preparation for the fourth chapter.

Students must be able to distinguish between a *thing* and the *names* we use for it. This distinction is most easily made in ordinary English where we can make statements such as "Canada has ten provinces" and "Canada has three syllables". We all understand that in the first sentence we are using the word 'Canada' to refer to the country while in the second we are referring to the word itself. It is customary to use single quotation marks to indicate that we are referring to the word. The second sentence should read, 'Canada' has three syllables. As a transition to the more difficult distinction between object and thing in arithmetic, the teacher might well use another example from English in which the thing is abstract. For example, he might assert that "life is short but art is long" and analyze the sentence. Are we referring to the words or things?

It is now standard practice on this continent to call the objects of arithmetic *numbers* and the labels for numbers *numerals*. Just as in English we have *letters* which form *words* which name *things*, in arithmetic we have *digits* which form *numerals* which name *numbers*. In the symbol '23', the '2' and the '3' are digits; '23' is a numeral. It names the number twenty three. The difficulty with this notion is that while we can point at a dog while saying 'dog' and thus distinguish between the animal and its name, there is no such easy way to point at twenty-three while pronouncing its name. Twenty-three is an idea in our heads. While this is a more difficult idea, it is quite necessary that it be understood. It is one of the small number of key principles lying behind all arithmetic. It is a good plan to give exercises such as this:

Sample exercise. *(Answers are given to the right)*

Which of these are properties of 24 and which are properties of '24'?

a	It is twice twelve.	24
b	It ends in '4'.	'24'
c	It is smaller than '16'.	'24'
d	It is larger than 23.	24

The teacher will immediately think of other ways in which this idea may be explored.

The teacher should take care that sentences such as " '37' ends in 7" are not accepted as correct. The quotation marks around '37' suggest that we are to examine the numeral. But the absence of quotation marks on 7 indicate that the symbol is being used to talk about the number seven. A collection of symbols such as '37' cannot end in a *number,* they can only end in another symbol. The sentence should read, " '37' ends in '7' ".

Notice that we usually wish to talk about numbers, not numerals. There is a very real danger that the teacher and the class will carry too far the numeral-number distinction and use the word 'numeral' in many places where 'number' is correct. For example, we do not add numerals, we add numbers. It is true that we manipulate numerals to find the sum of the numbers, but that is incidental. Arithmetic is performed with numbers, not numerals, and it is quite proper to refer to the numbers while manipulating numerals.

While this distinction is useful here, and aids in understanding, it is far more important later in the year when the fractional numbers are studied. The teacher of the inquisitive class might discuss the same distinction among those numbers at this point. For example, '$\frac{1}{2}$' and '$\frac{2}{4}$' are different names for the same number. This matter will, of course, be discussed further at the appropriate point in this manual.

In short, *the number-numeral distinction is important here and in further work, but it ought not to deteriorate into a verbal fussiness which impedes rather than aids understanding.*

BRACKETS ARE USEFUL (Pages 8 and 9)

Traditionally, students of arithmetic memorized rules of precedence in working with expressions such as $6+4\times7$. Because these rules of precedence are of no further consequence in mathematics, there is a growing tendency to use brackets instead in this kind of expression. This text provides both, and discusses brackets first so that this simpler way of indicating precedence may be used to discuss the more traditional rules.

The text maintains the practice of presenting correct terminology. At the same time, it should not be understood that memorizing the names of different kinds of brackets is of great consequence. Braces should probably be distinguished, because they are used to label sets.

EXPANDED NUMERALS (Pages 9 and 10)

This material provides a convenient bridge between the ideas of this chapter and those of chapter four. Students who have earlier worked in **Patterns in Arithmetic** should require only a very brief review here. This material should also be used to indicate how a number may be named in yet another way. Students who are unfamiliar with this notation should master it thoroughly as a preparation for later work.

NUMBER SENTENCES (Pages 11 and 12)

Throughout **Contemporary Mathematics I** and **II**, the idea of a number sentence is used to help clarify the relationships which may exist between numbers. This is a fairly recent technique which has proven very useful, particularly in discussing inequalities and open sentences, which will be discussed presently.

The signs '$=$', '$>$', '$<$', and '$\neq$' may be thought of as verbs. There are, of course, other mathematical verbs which will be introduced in later years. As a preparation for work with open sentences in Chapter 2, it is a good plan to discuss false sentences. That is, a class might consider the sentence, 'There is ten feet of snow outside.' The sentence may be grammatically correct, but be *false*. In the same way, the sentence $4-3=8$ is a mathematical sentence. It happens to be false. Paragraph six of this section may be generalized into an interesting kind of exercise. Students might be asked to find the missing operators in an expression such as

$$\frac{(5?8)?10}{2}=2$$

A more difficult kind of exercise involves placing the brackets too.

THE NUMBER LINE

GRAPHING ON THE NUMBER LINE (Pages 12 to 18)

Students who have worked in **Patterns in Arithmetic** should review this section quickly. Students who have not worked with the idea of using a number line as a model for the whole numbers should be exposed to the idea, but the teacher should use some caution. Classes who have not encountered this idea before occasionally resent what can seem to be a time consuming and needless elaboration of something that they already understand. The number line is introduced here as a convenient model on which students may test propositions about the rational numbers later and to lay the intuitive foundations for coordinate geometry in later grades. It would be unwise to do more than be certain that each student realizes that the number line *is* a model for the whole numbers and that he can graph sets of numbers on it.

ADDING

MULTIPLYING

SUBTRACTION

DIVISION (Pages 18 to 27)

These sections should not be regarded as attempts to reteach the operations of earlier grades. It may be that some students require reteaching at this point, but for most students in the seventh grade, such reteaching is properly regarded as being beneath them. At this level, the teacher will find that such matters are more effectively handled in some other context. Students will reveal arithmetical weaknesses in their regular work. Those that are not corrected in the large amount of exercise material provided in the text are a matter for individual diagnosis and remedy.

These sections of the text have quite another purpose. The teacher who is unfamiliar with the modern development from arithmetic into high school algebra should at this point read chapter two of the text closely, with particular attention to pages 66 to 76. In these pages a set of underlying principles are exposed, which, together with the open sentences also studied in the second chapter, will form the intuitive base for all algebra.

As these principles are studied, the student begins the transition from the naive view of the operations first given in the primary grades to the view that they are completely governed by a set of abstract principles. In these pages, a beginning is made towards this more abstract view of operations. In adding, the teacher should notice that the elements of two different sets are combined and then counted. The two addition examples are intended to have the student *consciously contemplate* an idea that he has probably unconsciously used for years, that the order in which whole numbers are added is irrelevant. The idea is not named here, but it is introduced.

In the same way, the notion that addition is associative is explored on page 20. While this principle should not be named at this time either, the teacher might well have students attempt to express the idea in their own words. The key idea is that this is a *generalization* about adding whole numbers. We are making a statement that is true for any three whole numbers.

The section on multiplication is intended to do for multiplication what the preceding section did for addition. Multiplication is introduced intuitively in the way in which it is most often applied, and this intuitive introduction is used as a basis for the discovery that multiplication is commutative and associative. The teacher should note that neither of these properties of multiplication is at all obvious. They only appear obvious to those who have been brainwashed to accept that they are true. It is probably best to teach this section by noting that it will show *why* something the class has always accepted as being true *is* true. The point should not be laboured. Classes which have become accustomed to exploring the ideas of arithmetic in past years will accept this material easily, but they will already have explored it. Classes which have previously done arithmetic in a "shut up—do this" fashion tend to resent this sort of material at first. Their common reaction is "So what. Tell us what to do". Since this idea will be developed thoroughly in the second chapter, it might well be referred to but not stressed here with such classes.

The short section titled "Adding and Multiplying" states the generalizations of the preceding sections in algebraic form. It should be omitted with classes which are not yet ready for such an abstract statement of the ideas involved.

The section on subtraction introduces the important relationship between subtraction and addition. It is most important that this relationship be thoroughly examined. The note in red on page 25 should be regarded as the teaching objective of the section. By a historical accident, the procedure for more complex subtraction used traditionally on this continent is one which does not tend to emphasize this relationship between addition and subtraction; in fact it tends to obscure it. At this point the teacher should have all students work a number of exercises in which addition or subtraction statements are written in other forms. For example, $9+4=13$ may be written as $13-9=4$ or as $13-4=9$. $17-3=14$ may be rewritten as $14+3=17$ or as $3+14=17$. A thorough familiarity with this idea here will pay considerable dividends for the next two years. While the section states that addition and subtraction are *inverse operations*, and the student might well know this terminology, the teacher should guard against a glib repetition of this phrase as a substitute for an understanding of its meaning.

The statement that division and multiplication are inverse operations might well be explored in a parallel fashion. Once again, our ordinary procedures for finding answers to division questions do not tend to emphasize this relationship. The discussion of "doing" and "undoing" operations is for heuristic purposes only. The essential understanding comes with the ability to recast a division sentence as a multiplication sentence or a multiplication sentence as a division sentence.

An understanding of these two relationships is important for arithmetical purposes. For example, most checking procedures depend on it. But they are even more important as a basis for the solution of the simple equations to be studied in **Contemporary Mathematics I** and **II**.

When algebra is studied formally in the ninth grade, formal equation-solving techniques will be mastered. In the meantime, these relationships provide a more limited but mathematically correct means of working with some equations.

For example, $9+n=17$ may be rewritten as $n=17-9$, $7n=42$ as $n=42\div 7$ and even equations such as $3n+11=29$ may be attacked in two steps.

The teacher should definitely not show this application of these relationships here, and not until chapter II of the text is studied at the earliest. Chapter II is intended to build up a fund of experience with open sentences to which the student may bring ideas such as these.

ZERO

UNITY (Pages 27 to 30)

The teacher who is familiar with high school algebra will recall that there are many places in that subject as well as in the arithmetic of the sixth grade in which the properties of zero and one are used to facilitate calculation. Those instances in which these properties are useful in this course will be discussed in context. For the time being it is sufficient to note that the intent of these two sections, as of the immediately preceding sections, is to establish *consciously* what has probably been accepted intuitively for some time.

Paragraphs 8, 9 and 10 of the section on Zero merit some discussion. It is essential that the student realize that questions such as $6\div 0$ are not just being postponed to some later date as are questions such as $8-13$. By the seventh grade, most students are aware that there *are* negative numbers. They realize that while a question such as $8-13$ is held in abeyance, it may have an answer. $6\div 0$ has no answer. The teacher might well use an example such as, "Where is five miles north of the north pole?" to make the point. All of the words of this sentence make sense, but the question has no answer. There is simply no such place. In the same way $6\div 0=n$ or alternately $n\times 0=6$ can have no answer. No matter what number we write for n, $n\times 0=0$. It is worth noting here that a misunderstanding which dates back some three hundred years (and which was resolved over 100 years ago) is still occasionally found. There is no number "infinity". $7\div 0$ does not equal "infinity" because there is no such number anywhere in mathematics. The symbol "∞" is used when limits are studied, but it is used in quite a different sense and never suggests a number.

$\frac{0}{0}$ or $0\div 0$ is less easy to dispose of. Many students seem to feel that it ought to be 1. It is first of all necessary to dispose of that possibility by noting that in $0=n\times 0$, n may be replaced by any number at all. If we were to accept these answers we could write $\frac{0}{0}=1$, $\frac{0}{0}=2$, then $1=2$. To prevent this we refuse to accept any answers for $0\div 0$. We say that $0\div 0$ is undefined. This argument is omitted from the text because it is felt to be a cumbersome argument at this grade level for such a small point. With most classes, it is probably best to dismiss $0\div 0$ as having no meaning.

CLOSURE (Pages 30 to 33)

This section does two things, one of which might be missed. The teacher is reminded that in mathematics, each number system is formally regarded as an entity in its own right and that similarities are noted afterwards. This notion is abetted here by thinking of closure in a *set of numbers.* The idea that we may select the set of whole numbers and then ask whether or not all questions have answers guides the student intuitively towards the modern view of a number system. It is an important part of this section that students note that the whole numbers are not closed under subtraction and division. It is traditional to speak of a number system as being "under" an operation.

This foundation will be particularly valuable in the eighth grade when the integers will be studied.

TEACHER'S NOTES

TEACHER'S NOTES

2

Open Sentences

Since about the first grade students have, by intuitive means, thought of number sentences which have "holes" in them. Their first experiences with subtraction at least made them think in terms of $4+\square=6$, even if they do not write the idea out in this explicit way.

In this chapter, students begin to explore open sentences explicitly. The means of exploration used is of quite recent origin. The great success which it has enjoyed whenever it has been used sensibly guarantees that it will form an important part of the transition from arithmetic to formal algebra.

Open sentences constitute an important bridge between arithmetic and problem solving, and are an excellent vehicle for the formal establishment of the fundamental principles of arithmetic that later become the fundamental principles of algebra.

Students who have worked with **Patterns in Arithmetic** or with some other good modern series will have become accustomed to writing open sentences for problems prior to this grade. Other students may meet this idea for the first time here. This chapter is written so that both groups of students may be served, but students with a weaker background will, of course, require more work. Students who have already written open sentences will learn to do so in a more general way than ever before in this chapter. They will take an important step towards what is an important part of any ninth grade algebra course.

Most students will not previously have used open sentences as a means of exploring the properties of the numbers of arithmetic. An attempt has been made, at the appropriate places, to give in considerable detail the intent of each such section of this chapter.

NUMBER EXPRESSIONS

PRACTICE IN EVALUATING NUMBER EXPRESSIONS (Pages 36 to 40)

It is intended here that the student will become familiar with the idea that the box in an expression such as $6+\square+4$ is a placeholder. We may replace the box by any numeral we please. If the teacher feels that the class requires further experience of this idea, the exercise on page 41 may be repeated with different replacements. This has the added advantage of stressing that numerals besides those given may replace the box.

There is a matter of language which the teacher should keep clear in his own mind: because a sentence is composed of words, it contains numerals, not numbers. An open number sentence is then made true or false by writing a *numeral* for the frame.

On the other hand, the open sentence refers to a relationship between numbers, not numerals. For this reason, and because it avoids cumbersome circumlocutions in the following discussion, it is customary to speak of a *number* as making the open sentence true or false, and, in this section, to speak of the replacement set as a set of numbers.

With more able classes the teacher may wish to discuss this problem and have them consciously agree to use the above convention. Other classes would probably see little point in the distinction between numbers and numerals in the language used and it causes no problems.

The teacher should also note that a frame is *not* a numeral. It does not name a number. He may wish to call frames pronumerals, by analogy with pronouns, but the analogy is probably lost on students who do not have a strong grasp of grammar.

THE REPLACEMENT SET (Pages 42 to 44)

The idea of replacement set is quite important in later work. For example, whether or not there is a solution for $3 \times \square = 5$ depends upon the replacement set. If the replacement set is the set of all fractional numbers, then there is a solution. If the replacement set is the set of all whole numbers, then there is no solution. In the exercises on page 43, the teacher should be careful to distinguish between proper and improper replacements on the one hand and true and false replacements on the other. In the first example, 'Chicago' is an improper replacement. The fact that it would make the sentence true is beside the point. "Ottawa" is a proper replacement that makes the sentence false.

The teacher should note that the replacement set for the last exercise of this section uses the whole numbers from 0 to 30 inclusive as the replacement set. It will be unusual to work with such restricted replacement sets. The teacher should note, but not teach formally, several properties of open sentences established in this exercise:

a *Open sentences are not the same thing as equations.* For example, $2+2=4$ is an equation but not an open sentence, while (a) of this exercise is an open sentence but not an equation.

b The numbers which make an open sentence true are, on page 55, identified as the solution set. For the time being the explicit label "solution set" may be avoided. *Students should be less concerned with vocabulary than with ideas at this point.*

c A most important way of looking at equations is established here for the first time.

It has already been noted that in the open sentence $\square + 13 = 29$, which will later be written as $t + 13 = 29$, the box or the t is not a cryptic symbol for a particular number that we have not yet identified. The student is not apt to have as much difficulty with this as the teacher who learned to think of letters in that way in his own high school algebra.

The difficulty with the traditional view may be seen in two open sentences:

$$\square+2=\square+3 \text{ and } (\square\times\square)+6=5\times\square$$

Both of these open sentences are somewhat difficult for this exercise, but are studied later. It does not make sense to think of the box as representing a particular number in the first sentence. There is no such number. Yet it is a perfectly correct open sentence. It merely turns out that the set of numbers which make it true is the empty set. The second open sentence has two different numbers which make it true. If $\square$ represents some particular number, we are again in difficulties.

The teacher should take care that his own language emphasizes these characteristics of the placeholders in open sentences.

SEQUENCES (Pages 44 and 45)

It is difficult, and often impossible, to discuss the full solution sets of important open sentences without displaying infinite sets of numbers. This section extends the earlier work with '. . .' notation. Here '. . .' is used to represent missing numerals in a list and also to express the earlier meaning.

The discussion in paragraph 3 is quite important. It will later be the motivation for writing some precise algebraic *rule*, stipulating the way in which further elements of the sequence are to be found. The most able classes might discuss this problem here.

NATURAL NUMBERS AND WHOLE NUMBERS IN OPEN SENTENCES (Pages 46 and 47)

The discussion on page 46 suggests the way in which one might attempt to find the set of numbers which makes an open sentence true. The following exercises are very important, and the teacher might well choose to give additional exercises at this point and use open sentences of this kind in quick drills throughout the year.

It is of paramount importance that the teacher does not suggest any rules by which solutions might be found or teach any rituals for simplifying open sentences.

These exercises are *not* formal exercises in high school algebra in which boxes replacing x's. The student is expected to meet each new open sentence as a problem, and is expected to use his intuitions about numbers to solve them. Some students will find some of these exercises quite difficult. These will be students who have not developed necessary understandings about numbers in earlier grades. Any provision of procedures for solving equations by the teacher will completely frustrate the intent of this work.

To this end, questions such as, "How did you work that out?", "Did anyone work that out another way?", "What number did you try first?" and "What makes you sure that there are not other numbers that work?" should be quite common. *Formal language should be avoided as much as possible at this stage.*

Many students will apply their earlier study of the relationship between addition and subtraction and between multiplication and division to the solution of open sentences.

While this practice ought not to be discouraged and in fact demonstrates an important step in understanding for the student, the teacher must do everything possible to permit as many students as are able to discover this procedure for themselves. One of the best ways is to avoid explicit discussion of the use of these relationships until it is evident that most students are using them.

From that point on, it is still more important that students build intuitive understandings of open sentences than that they have formal means of solving them with dispatch. Such formal procedures are for the ninth grade, not the seventh.

It will be found that in some classes there is a great reluctance to try any number in an open sentence. Some children seem to be terrified of writing down anything which might turn out to be wrong. It is sometimes necessary to go to some lengths to incite students to begin attempting numbers. One useful lever is to ask, "How old are you?" and to the answer say, "Put *that* down, try 12", or whatever the student says. Many students who have learned all previous arithmetic as a set of meaningless rituals here, for the first time, begin to see the structure of the subject. Others will move on to more difficult problems and develop their insights further.

Once again, it is important that the teacher be familiar with the material on pages 66 and 76 while working with these open sentences. Many of these open sentences involve ideas that are met formally there.

USING OPEN SENTENCES TO SOLVE PROBLEMS (Pages 48 and 49)

From the first grade, students solve problems by inspection. On the playground they perceive a situation involving marbles or members of a team, and without conscious thought move to an arithmetic solution. In the classroom, they have solved them in the same way, except that after solving them they have had to write statements of some sort to summarize the problems and the solutions.

When problems are quite simple, it is appropriate that they be solved in this way. As they become more difficult, generally at about this stage of mathematics, it becomes useful to solve problems in two stages:

a Translate the problem into an abstract pattern such as an open sentence.

b Solve the open sentence.

There comes a time, within about two years, when the student has no choice but to solve problems this way, because the relationship between the numbers of the problem is too involved to be handled completely in the mind.

In order to facilitate this transition at a later date, most modern arithmetic programmes begin in the primary grades to suggest simple open sentences for problems. For students who have experience with such an earlier programme, this section will constitute a minor extension of what they already know. Students who are not already familiar with this notion should study this section closely.

The examples given are stripped of the usual concrete clothing of a problem so that such students may concentrate upon the relationships between the numbers. There is no urgency in developing ease with this way of representing problems.

Contemporary Mathematics II builds on this introduction and takes another step towards the general writing of equations for problems involving the real world.

In later chapters of the text, problems of all kinds are introduced in context. Generally, no technique of solution is mentioned. With all classes the teacher should make it a practice to write open sentences for almost all problems.

Where the teacher does not wish to have a problem written up formally, it is a good plan simply to read the problem, agree upon an open sentence that is then written on the chalkboard, and find the solution set for the open sentence.

EVALUATING NUMBER EXPRESSIONS (Pages 49 and 50)

In this section, the notion of two different placeholders is introduced. A certain simplification of language is introduced here which the teacher should recognize. If we are to be quite correct, we must say that in an expression such as □+△, we will let 4 replace the triangle and 6 replace the box and then evaluate the resulting expression. Such a statement would be very cumbersome. Instead we say, "Evaluate □+△ if □=6 and △=4." Strictly, □=6 and △=4 are open sentences and we must *still* decide that 4 replaces the triangle in the second open sentence.

A box or a triangle can never be a numeral, it is a placeholder.

This distinction has not been introduced in the text, because explicitly noting it would probably confuse rather than clarify the issue. The teacher can keep the matter clear by always reading these problems in this way:

Text	*Teacher*
Evaluate □+2×△for □=5, △=2	Evaluate □+2×△ if we let 5 replace the box and 2 replace the triangle.

Most of the questions in this exercise may be repeated for any other values of the placeholders. In a few cases some care must be exercised in selecting other replacements.

PAIRS OF NUMBERS IN OPEN SENTENCES (Pages 51 to 55)

Notice that while different numbers *may* replace each of two different placeholders, these replacements need not be different. The exercises on page 53 and 54 bear the same injunction as the earlier exercises involving open sentences. It would be a serious error to provide any more aids to finding appropriate pairs of numbers than are given in the text. *One of the main intents of these exercises is that students should discover for themselves important underlying relationships between numbers.* It is an interesting and significant task to spend one period discussing the solutions for the open sentences on page 54 and possible variations of them.

The replacement set and the solution sets are discussed in the most natural way possible. A precise discussion of the replacement set and the solution sets for these open sentences involves concepts which are not appropriate to the seventh grade. It is sufficient that the teacher note that while the replacement set is given as being the set of whole numbers for each placeholder, the solution set consists of *pairs* of numbers. Brighter students may notice that this is strictly speaking incorrect: if the solution set is to be drawn from the replacement set, then the replacement set should also consist of pairs of numbers. Any correct discussion of this matter involves, in some form, what is called the Cartesian product of two sets. This notion is certainly far beyond the needs of most seventh grade classes.

There is no need to raise this matter at all unless it is raised by a more able student. In that case, the teacher should agree that the replacement set for an open sentence with two variables consists of pairs of numbers.

The problems of paragraph 9 isolate the problems of attaching open sentences in two variables to situations in the real world. Some students may be concerned that a numerical solution is not found for each problem. It is interesting to attach some further information to each problem after the open sentence is found. For example, in the first problem, the teacher might ask what Mary's allowance would be if Joe's allowance were 35¢. A few such examples should help students understand that the open sentence is the solution to the problem as stated and that further information would make a definite numerical solution possible.

THE SOLUTION SET (Pages 55 to 57)

The first part of this section reworks some earlier material, but using the more formal definition of a solution set. *The teacher should take care that in this section and subsequent sections the language introduced here is used.* Note how appropriate the language of sets is to this discussion.

In the last part of this discussion there is a further use of open sentences to analyze problems. In these problems are written open sentences that must be solved by inspection. For example, in problem *a*, after writing $(4\times\square)-31=125$ it is simply asserted that 39 must replace the box. In a text, the answer must either be provided or not be provided. There is no way to suggest that the number was found by trial. It would be worthwhile to work this problem on the blackboard and solve the open sentence by trial before the text is opened. In any case, *no formal means of solving equations should be introduced here.* The teacher will note by this time that some students are beginning to systematize their explorations. For example, in $(4\times\square)-31=125$ some students will immediately note that $4\times\square$ must be 31 more than 125, and will add. Others will use the fact that subtraction and addition are inverse operations. This is, of course, excellent, and these students have already laid the foundation for sound equation-solving skills, but the students who have not yet come to such general techniques should persist in working by trial and error. *There is no hurry in the seventh grade to achieve formal equation-solving skills and the student who is given formal rules in advance of his understanding here will be cheated of one of his last opportunities to come to understand these relations intuitively.*

USING NUMBER SENTENCES TO SOLVE PROBLEMS (Pages 58 to 60)

This section gives further practice in writing open sentences for problems. All of these problems involve open sentences which are equations in one variable.

USING LETTERS IN OPEN SENTENCES (Pages 61 and 62)

Students who have worked with **Patterns in Arithmetic** are already accustomed to using both letters and frames in open sentences. Those students for whom this is the first exposure to open sentences will require some motivation for changing from frames to letters. The explanation given on page 61 is the correct one: with several different variables, it becomes far more convenient to use letters.

The teacher should bear in mind that with students who have not met this notation before he must re-inforce at this point that the letter is still a placeholder, not a cryptic symbol for a number. The letter may be *replaced* by a numeral; it is not itself a numeral. Once it is replaced by a numeral representing a number in the replacement set, the sentence will become true or false.

The following exercises provide experience with this new notation and additional work with open sentences.

ANOTHER LOOK AT SOME NEW SYMBOLS (Pages 63 to 65)

The symbols > and < and the more important convention regarding multiplication signs are introduced, the inequality signs for the second time. Once again, the exercises give experience with the new notational agreements and further work with open sentences. As with earlier examples, some time should be given to a discussion of how solutions are reached as well as to solutions themselves.

MORE ABOUT THE PROPERTIES OF NATURAL NUMBERS (Pages 66 to 76)

This section unites two earlier important lines of study. In the first chapter of this text there was some intuitive exploration of the commutative and associative ideas. In previous sections of chapter 2 open sentences were worked with directly. In this section, open sentences are used to state the fundamental properties of the natural and whole number systems. Formal language is introduced in context.

While the ideas behind this chapter are intuitively obvious, the teacher should not underestimate the need for careful teaching. Contemporary algebra courses are invariably built completely on these principles. It is not sufficient that the student say, "Oh yes, I understand that". *It is necessary that these principles become second nature to every student. In all following work, the teacher should seize on every opportunity to use the vocabulary and the ideas of this section.* From this chapter on, and throughout the eighth grade, arithmetic will never be the same. Whenever it is studied, it should be studied in such a way as to exhibit these principles. Each principle is introduced carefully with ample immediate exercise material, but these topics demand continuing attention throughout the year.

The first paragraph notes that an open sentence which becomes true for every replacement must express some underlying property of the whole numbers.

The first principle introduced is the *commutative property of addition.* Through physical and numerical examples, the student is led to attach all of his earlier intuitive feelings about this notation to the word *commutative.* The teacher should note that the physical examples given are merely heuristic. They are mostly open to argument. For example, the actions of going five miles north and then five miles east are only commutative if we are strictly concerned with the final point reached and if we are on a flat surface. These trips may not be commutative in fact. Such examples are probably most instructive when they illustrate situations that definitely are not commutative.

The questions in exercise 6 are not intended to involve *only* the commutative idea. From a formal point of view, in which every operation of arithmetic must be justified by some principle, these questions are quite complex. That is not the intent of these questions. Students are here discovering principles in a body of material with which they have long experience. Seventh grade students are not sufficiently mature to accept all the strictures of a purely deductive approach to the whole numbers. It is intended in this exercise that they will use all of their accustomed procedures for adding, but may also consciously use the commutative principle to facilitate calculation.

Once it is established that multiplication is commutative, a similar exercise is presented. Once again, it should be noted that the student is expected to use all of his past procedures for multiplying, but may now consciously use the commutative principle as well.

The commutative idea is quite easily established. The associative idea is more subtle, and the teacher should expect some difficulty with it, particularly with slower students. It is particularly useful to begin with an example such as that given at the top of page 70. It is worthwhile working many such examples with slower classes. A whole set of such questions might be given, with instructions to work each question as many different ways as possible (disregarding the conventional rules of precedence.) The occasional expression such as $4+6+3$ and $8\times6\times2$ may be included in such an exercise.

The essential outcome of such an exercise is that every student notes with some slight surprise that there is something peculiar about the above two situations; a general caution about rules of precedence or brackets may be safely ignored here. Most teachers who have taught the associative principle at this level have found it better to have the class think of the expression as pre-existing and the brackets as being added than to start with an expression such as $(5+3)+9$ and then think of moving the brackets. There is no motivation for this latter exercise.

Some students will confuse the commutative and associate principles if care is not taken to impress upon them that whenever we use the associative idea by itself, no changes of order are ever involved, only changes of grouping.

The material of the exercise at the bottom of page 71 is particularly useful to help give students an intuitive notion of the associative principle of addition.

It is advisable to have the associative idea of addition thoroughly understood before introducing it in multiplication. The same sort of exercise is used. The resulting shortcut for some multiplication questions is worth learning.

While the previously mentioned principles are important, the last to be studied here is certainly the most important for further work. This is probably because, being more complex, it is more often necessary to use it explicitly. The quite lengthy development in the text leads to a statement of and an understanding of the principle embodied in the two open sentences

$$(a+b)n=an+bn$$
$$\text{and } n(a+b)=na+nb$$

While these open sentences are convenient for expressing the distributive principle, they do not express well the way in which it is used. This is because of one important difference between open sentences and statements in arithmetic. When we write 14×6, we can immediately think, $(10\times6)+(4\times6)$. We have used the distributive principle. Yet there is no equivalent way to express what we have done using an open sentence, because an expression such as $(a+b)$ cannot be combined into one letter. For this reason, the teacher should take care that the examples and questions involving multiplication of whole numbers are studied thoroughly. It is worthwhile giving questions of the kind in paragraphs 22 and 24 throughout the year.

The teacher will recognize that this principle becomes the basis of much algebraic work later, including almost all factoring.

The brightest students may go on to study what is sometimes called the double distributive situation. A question such as $23\times17=(20\times17)+(3\times17)$. Then each of the terms on the right may in turn be broken up.

$$=[(20\times10)+(20\times7)]+[(3\times10)+(3\times7)]$$

While this exercise is probably best reserved for brighter students in the seventh grade, it should not be regarded as being frivolous. If it is once mastered, it can be used to advantage later multiplying mixed numbers and is of course of considerable importance in algebra.

MULTIPLICATION AND DIVISION (Pages 77 to 80)

This material is purposely delayed until after the study of the principles. In addition to those given in the text, the teacher will note many instances in which some procedure for finding an answer is illuminated by one of the principles. For example, in the division questions, it may be noted that

$$(100\times247)+(30\times247)+(5\times247)=135\times247$$

Students who have learned multiplication in any modern programme will be acquainted with the way in which multiplication questions are set out here and will see in this form an application of the distributive principle. Students who do not have this background should use this material to see how the algorithm that they may have learned as a less meaningful ritual is really developed. Some teachers like to suggest that students eventually leave off the "superfluous" zeros at the end of each line. There is no good reason why this should be done, and we recommend that they be left. Removing them serves no good purpose and might well lead to errors.

There is no doubt that the subtractive form of the division algorithm is better for teaching and understanding purposes than the traditional form. But in this case, the traditional form of the algorithm has something to recommend it in economy of writing. It has been the policy throughout **Patterns in Arithmetic** and **Contemporary Mathematics** to use the subtractive form of the algorithm for teaching purposes, but increasingly to show that the algorithm may be summarized in the traditional short form for computational purposes. This policy is maintained here. These algorithms are not regarded as being independent, but as a full and short form of the same procedure. Whenever some new idea is to be understood, the full form is to be preferred.

This short section reviews both forms of the algorithm and provides some examples. If further examples are required, they are readily constructed.

It should be noted that in these questions all remainders are left as remainders. For checking purposes, an example such as:

$$\begin{array}{r|l} 247\overline{)33355} & \ldots\ldots \\ \ldots & \ldots\ldots \\ \ldots & \ldots\ldots \\ \hline 10 & 135 \end{array}$$

may be rewritten as $(135 \times 247) + 10 = 33355$

Short division is introduced in the seventh paragraph of this section. While the questions in exercise 9 will be sufficient to enable most students to master the idea, further exercises should be given from time to time.

TEACHER'S NOTES

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3

Factors

Most teachers at the seventh grade level are impressed with the need for constant application of arithmetic. But when this takes the form of constant repetition of simple questions involving the four fundamental operations, the arithmetic period becomes dull and routine. Furthermore, the lack of motivation in the material tends to make students repeat their tasks in a routine desultory fashion, with no real gain in either proficiency or understanding.

The material of the third and fourth chapter of this text is ostensibly new material. These chapters can lead to new and important understandings of the whole number system. In the case of the third chapter in particular, the understandings developed here are of immediate and sustained value.

At the same time, both of these chapters demand the performance of large quantities of arithmetic of all kinds. When it is performed in a new and interesting context of this kind it is far less likely to be considered repetitious or dull.

FACTORS AND MULTIPLES (Pages 83 and 84)

The definition of 'divides' given here has long been the mathematical one, but is at variance with some traditional school usage. The teacher should take care to note that 'divides' here and henceforth means 'divides exactly'.

PRIME NUMBERS (Pages 84 to 86)

Note that the definition of prime numbers given here excludes 1 but includes 2. By this definition, 1 is neither a prime nor a composite number. Some students will feel that 2 is not really a prime number; after all, all the other primes are odd. The teacher should dispell this notion. It is true that 2 is the only prime divisible by 2, but how many primes are divisible by three? Clearly 3 is the only prime with *that* property. In fact, the peculiarity of 2 is the peculiarity of *every* prime number.

The word 'factor' is used in two senses in this section. When used as a noun, the word 'factor' means a number which divides a given number; viz. 7 is a factor of 21. When used as a verb, the word 'factor' means to write as a product of smaller numbers. Since we rarely need to use the expression any other way, we usually take the verb 'to factor' to mean to write as a product of *primes*, although this is not strictly necessary. In the text, the headings will be precisely correct, while the looser usage will generally be adopted in discourse.

FACTOR INTO PRIME FACTORS (Pages 87 and 88)

At first sight, this section seems an unnecessary intrusion of more advanced material into an elementary discussion, but it should not be omitted. More able students, particularly, often worry about the order in which they will remove factors when they simplify fractions. They are often found to have the suspicion that removing the wrong factor first might 'jam things up', while starting with the right factor might lead to great simplification. The exercise at the bottom of the page, while brief, is sufficient to establish the power of the theorem.

COMPOSITE FACTORS (Pages 88 to 92)

The material of this section demands large quantities of division and multiplication, and promotes a facility with the notion of a factor. The teacher will notice ample opportunity to use the words *commutative* and *associative*! It is particularly easy to construct further exercises like number 7. It is necessary only to select two or three small numbers, each with few, if any, prime factors, multiply them, and present the product. Numbers with large numbers of factors probably teach little and lead to boredom.

USING FACTORS TO DIVIDE (Page 92)

It is not suggested that students use the method of division of this section for ordinary division. There are special situations, however, in which it might well be used. In particular, it commends itself to those cases where both the divisor and the dividend are multiples of ten.

COMMON FACTORS (Pages 93 and 94)

The teacher should notice that the language of sets is again particularly appropriate for the discussion of common factors. The notion of the intersection of two sets, not mentioned explicitly earlier, may well be suggested here, in an informal way. The intersection of two sets is the set of elements common to both sets.

The kind of question in exercise 4 of this section should be worked until the idea of a list of common factors is established.

GREATEST COMMON FACTOR

RELATIVELY PRIME NUMBERS (Pages 94 and 95)

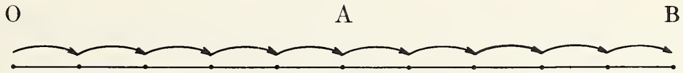
The conclusion grows directly out of the immediately preceding work that each list of common factors of a pair of numbers must have a greatest number. When this number is 1, the numbers are said to be relatively prime. These sections of this chapter have been kept quite short because, while they do present an interesting vehicle for multiplication and division, slower students may resent the abstract nature of the discussion.

With average and brighter students, there are some interesting side-problems dealing with greatest common factors and relatively prime numbers. Given any two relatively prime numbers, there is always a pair of multiples of the numbers which differs by one. For example, 18 and 25 are relatively prime numbers. $7\times18=126$ and $5\times25=125$. Students might attempt to find the smallest pair of such multiples for each pair of relatively prime numbers they study.

It is also true that any pair of numbers that differ by one must be relatively prime. Brighter students might attempt to show why this must be so as a prelude to the next side-topic.

Euclid's algorithm was omitted from the text only for the reasons mentioned earlier. Like the more directly applicable work included in the text, it is both interesting and involves considerable division.

Euclid's algorithm provides an alternative method of finding the greatest common divisor of a pair of numbers. The basic proposition to the algorithm is that any divisor of two numbers must divide their difference. This may be readily demonstrated on a number line.



The multiples of the divisor may be shown as 'jumps' on the line. It is given that this particular divisor divides both A and B. In other words, it lands exactly on the points marked A and B. It requires but a moment's reflection to see that there must be an exact number of 'jumps' between A and B.

Since *any* divisor of two numbers divides their difference, in particular the greatest common divisor of two numbers divides their difference. This is not Euclid's algorithm, but it is the first step towards it. Some exercises may be performed at this level.

For example, we might find the greatest common divisor of 51 and 85 this way. The greatest common divisor of this pair of numbers must also divide their difference, 34. Since it divides both 34 and 51, it seems reasonable that it is also the greatest common divisor of 34 and 51. And then the game is on. The greatest common divisor of 34 and 51 must divide *their* difference, 17. And it must be the greatest common divisor of 17 and 34. And it must divide their difference, 17. Hence the greatest common divisor of the original pair, 51 and 95, is 17.

With a pair such as 12 and 78, it would be possible to replace 12 and 78 by 12 and 66, 12 and 66 by 12 and 54, 12 and 54 by 12 and 42, 12 and 42 by 12 and 30, 12 and 30 by 12 and 18, and finally 12 and 18 by 6 and 12. The greatest common divisor is clearly 6. But it is also clear that 12 is removed from the 78 and from each succeeding difference until the remainder is less than 12. In other words, we may find this remainder by division. This is Euclid's algorithm. The greatest common divisor of a pair of numbers is also the greatest common divisor of the lesser, and the remainder when the larger is divided by the lesser. This provides a very rapid means of finding the greatest common divisor of even quite large numbers. For example, the greatest common divisor of 345 and 1065 may be found as follows:

$$\begin{array}{r|l} 345\overline{)1065} & 3 \\ \underline{1035} & \\ 30 & \end{array} \qquad \begin{array}{r|l} 30\overline{)345} & 10 \\ \underline{300} & \\ 45 & \\ \underline{30} & \\ 15 & \end{array}$$

The greatest common divisor of 345 and 1065 is *15*.

Well constructed exercises give all students considerable practice in dividing. More able students may be encouraged to learn Euclid's algorithm partly because of its intrinsic interest and partly because of its value in more advanced algebra.

COMMON MULTIPLES (Pages 95 and 96)

The teacher should note that 0 is a multiple of every number. Its inclusion every time a list of common multiples is given is a matter of taste. Unlike 1 in the list of common factors, there is no immediate application of this fact short of university level number theory. In any case, it is necessary to exclude 0 when the Least Common Multiple is studied. For this reason the text points out that 0 is a common multiple of any pair of numbers, and then agrees no longer to list it.

As with the greatest common divisor, the longer list of common multiples is studied before the idea of a least common multiple is introduced.

LEAST COMMON MULTIPLE (Pages 97 to 100)

There is still in common usage an older algorithm for finding least common multiples. In worked out form, it looks like this;

$$\begin{array}{r} 2\underline{)36,\ 48} \\ 2\underline{)18,\ 24} \\ 3\underline{)\ 9,\ 12} \\ 3,\ \ 4 \end{array}$$

$$2\times2\times3\times3\times4=144$$

This is a fair example of what has been called "shut up—do this" arithmetic. The alternative algorithm presented by a knowledge of factors and primes is so attractive that it is suggested that the above algorithm should never be used.

Paragraph 2 of this section, it will be noted, does not involve using any procedure at all. Students are asked to find least common multiples by inspection. Only the following discussion suggests the way they may be found using the factorization of numbers.

There is another interesting side-trip which may follow paragraph six. Write factorizations of two numbers as follows;

$$\begin{array}{r|l|l} 180=3\times2\times & 2\times3\times5 & \\ & 2\times3\times5 & \times7\times11=2310 \end{array}$$

The product of 180 and 2310 will certainly be the product of all the factors shown. The least common multiple will not be that great. In fact, we can see exactly what the least common multiple does not have in common with the product. In this case it is the factors 2, 3, and 5. But these factors are just the greatest common divisor.

If we were to multiply the least common multiple of two numbers by their greatest common divisor, we would obtain their product. The relationship may be written as follows;

$$\text{L.C.M.} \times \text{G.C.D.} = \text{Product}$$

Of course, this relationship is of no value for finding the product of two numbers, but it is most useful for finding the L.C.M. The product of a pair of numbers is easily found, and by Euclid's algorithm the G.C.D. is easily found. The quotient of these two numbers must be the L.C.M.

Paragraph 7 extends the idea of least common multiple to three or more numbers. Once the basic idea is mastered, what remains is largely an arithmetic exercise. When more than three numbers are given the arithmetic may be considerable. The very bright grade seven student might attempt to extend the relationship between L.C.M., G.C.D., and Product given above to the case of three numbers and then more. This is an excellent exercise for such a student, and since it is an open-ended problem allows him to push himself to his limit.

EXPONENTS (Pages 100 and 101)

Treatment of exponents has been delayed until this point so that the language of factors and products may be used to discuss them in a compact way. The following section on square root, a natural application of factoring, also requires that this material be covered here. Notice that the word 'base' is introduced. The word is used in the same sense here as in the following chapter in which numerals to different bases are discussed.

TEACHER'S NOTES

TEACHER'S NOTES

TEACHER'S NOTES

4 Numeration Systems

ANOTHER LOOK AT THE DECIMAL NUMERATION SYSTEM (Page 107)

A thorough examination of the nature of place value forms such an important part of earlier texts in the **Patterns in Arithmetic** series and in most other modern series that the idea is mentioned here only as a reference point for the departure to another base, which follows.

Students who are not familiar with the idea of place value and expanded notation should not proceed until they are familiar with the material of this section.

ANOTHER BASE (Pages 107 to 112)

With this section there begins the main work of this chapter, the study of numeration systems to other bases. Here, more than in most other chapters of this text, the teacher must have foremost in his mind at all times the purpose of the chapter. It is possible to become so engrossed in the material of the chapter, or so intent upon solving specific problems, that the whole point is lost. It should be remembered that this is not part of a consecutive development. No further work depends directly upon the *results* obtained in this chapter.

The study of numeration systems to bases other than ten has been shown to be one of the most powerful techniques available to help a student understand the nature of our place value system. Our number system is studied from a very early age, and students often become as glibly familiar with it as they do with the English language before they are sufficiently mature to understand the way in which it is built. This is a right and necessary thing; no one would seriously suggest that young children should learn English by studying grammar or that they should learn about arithmetic by studying abstract mathematical rules; but there comes a time both in the study of language and in the study of arithmetic when further progress depends upon an understanding of the structure of the subject.

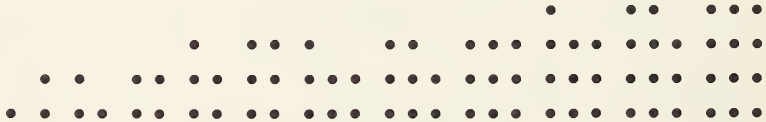
It is now the custom in most good primary and intermediate programmes to deal with place value in a more explicit way than was once the case. For such students this study of other notational systems will be a natural extension of what they already know. For other students this chapter may be a revelation. If the teacher keeps the objective in mind at all times, students must emerge from it with a much greater insight into our base ten system.

There are several ways of making convincing the notion of using another base. The approach here is historically convincing. People might have begun to count the way suggested here. The teacher with a class who prefer wilder images may profitably suggest that the possibility of making contact with extra-terrestrial life is now greater than ever before: a serious attempt to detect intelligent messages from outer space using a large radio telescope was made in the early 1960's. If such contact is ever made, certainly the first communications will involve numerals. There is no guarantee that such beings would use a base of ten. We could suppose that they had three fingers on each hand, or two on one hand and three on the other, or whatever we wish.

There are other approaches which involve putting books in packages of five, packages into cartons of five etc. Regardless of the approach, it should be noticed that the study of other bases should not begin with counting, but with representing quantities to other bases. The idea of counting is one of the more difficult ideas involved and is best delayed until that of representing quantity is established.

Some texts have refused to use ordinary numerals as subscripts. The authors have not found that using them leads to any confusion. Classes, even those of low ability, seem readily to accept the convention that subscripts are always in ordinary base ten notation. Thus it is possible to write 12_5 instead of the more cumbersome 12_{five}.

The first exposure to counting, in the fifth paragraph of this section, will probably be the first point at which the teacher will have any difficulty. Some students will persist in the notion that counting base five means leaving out all the numbers from 6 to 9. This is an excellent place to repeat the earlier distinction between number and numeral. It is a good plan to draw domino patterns of dots across the front of the room and label the quantity in each pattern, first using base ten numerals and then using base five numerals.



Base 10	1	2	3	4	5	6	7	8	9	10	11	12
Base 5	1_5	2_5	3_5	4_5	10_5	11_5	12_5	13_5	14_5	20_5	21_5	22_5

On such a diagram it is easier to talk about the *quantity* of dots and the various *numerals* we use to represent the quantity. Here, if not before, some students may really grasp the distinction between number and numeral. Notice that even the symbols 1, 2, 3 and 4 are given subscripts. These will be found useful as a reminder of the system being worked with. Throughout, the authors have adopted the convention that where subscripts are not shown base ten is implied. At some point in the development brighter students may notice that "ordinary" numerals should also carry a subscript. There is no harm using a subscript of ten for a time if this is desired. viz; 34_{10} instead of 34.

There are two ways in which the translation of base ten numerals in some other base may be developed. It has been the practice throughout **Patterns in**

Arithmetic and **Contemporary Mathematics** to give alternative meaningful developments wherever possible. In this case, both developments are meaningful, and one of the two was selected only because they are both lengthy. The development used here makes greater use of the idea of exponents, and for that reason probably serves more to integrate this unit with earlier ones.

The teacher should know the alternative development for more than academic reasons. There are always some students who seem to understand one development better than another, and who may respond better to the alternative development. It is also a generally accepted theory that students understand an idea better if they know two different approaches to it. Classes that have time might well profit from a study of the alternative development.

By the method of the text, 193 is translated to base five notation as follows:

$$\begin{array}{llll} \begin{array}{l} 5^2=25 \\ 5^3=125 \\ \\ \\ \end{array} & \left.\begin{array}{r} 193 \\ -125 \\ \hline 68 \\ 1 \end{array}\right) & \left.\begin{array}{r} 68 \\ -50 \\ \hline 18 \\ 2 \end{array}\right) & \left.\begin{array}{r} 18 \\ -15 \\ \hline 3 \\ 3 \end{array}\right) \end{array}$$

$$193=1233_5$$

First, the largest multiple of the largest possible power of 5 is subtracted from 193. Then the largest possible multiple of the next largest power of 5 is subtracted from the remainder, and so on. The text describes this process in detail.

By the alternative method, the entire group of 193 is packaged in fives. The calculation might be shown as follows:

$$\begin{array}{r} 5\overline{)193\quad} \\ 38+3 \end{array}$$

It is possible to obtain 38 packages of 5 from 193, with 3 left over. This "3" is recorded as the units figure of the translated numeral. The 38 packages are in turn put up in fives.

$$\begin{array}{r} 5\overline{)193\quad\;} \\ 2\overline{)38+3} \\ 7+3 \end{array}$$

There are then 7 "cartons" with three packages left over. This "3" is recorded as the "fives" figure of the translated numeral. The calculation is completed as follows.

$$\begin{array}{r} 5\overline{)193\quad\quad} \\ 5\overline{)38+3\;\;} \\ 5\overline{)7+3} \\ 1+2 \end{array}$$

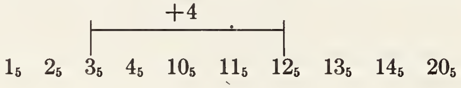
$$193=1233_5$$

Taught with sufficient care, this approach is as easy to understand as the approach in the text. It is a good plan to begin with small quantities, actually to draw them out or represent them as sets of objects on a table, and watch the formation of packages of various sizes.

The method of the text has the advantage that it immediately suggests the means of translating from some other base back to base ten.

OPERATIONS WITH BASE FIVE NUMERALS (Pages 113 to 118)

There are two ways in which students might complete the questions of the first paragraph of this section. The teacher should show both methods. For a question such as 3+4 it is possible to add the numbers as though numerals given were base ten numerals and then to translate the sum to base five. It is also possible to begin with a list of numerals base five, and count out the sum using the list:

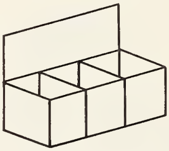

$+4$
$1_5 \quad 2_5 \quad 3_5 \quad 4_5 \quad 10_5 \quad 11_5 \quad 12_5 \quad 13_5 \quad 14_5 \quad 20_5$

If we begin at 3_5 and count ahead 4, we clearly end at 12_5. Since both procedures are so simple in this case, there is no reason why both should not be used.

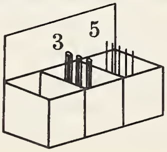
The table in the second paragraph merely summarizes the results of the first paragraph. The teacher should note that, by convention, the first given number is read down the side of the table and the second across the top. The distinction is not important in systems that are commutative. When the student studies systems that are not commutative or in which this property is in doubt (a characteristic of most contemporary ninth grade programmes) he will have established a consistent and correct habit.

The third to the twelfth paragraphs of this section develop the algorithm for adding base five. Here the earlier caution against losing sight of the original objective is particularly important. It is not difficult to teach children to "write down the two and carry one", or whatever the case may be. The intent of these paragraphs is that by thinking through the reasons why we may "carry" base five, the student will see the algorithm for carrying base ten in a new perspective.

Many classes can profit by the use of a device more commonly used now in the primary grades, the place-value box, which is easily constructed of plywood or cardboard if necessary.

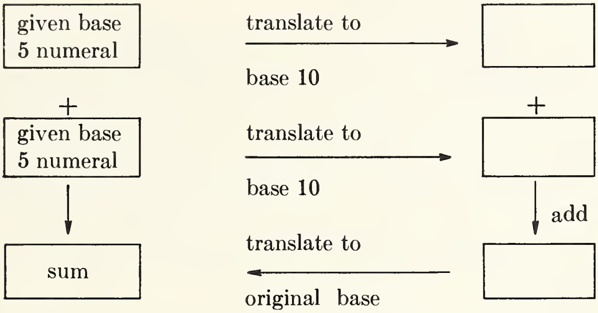


There are commonly three or four bins. It is a good plan to mount a small piece of blackboard on the raised back. This box is used in the primary grades to teach the idea of place-value for the first time. When objects such as coffee-stir-sticks are placed in the ones' bin, they are packaged in tens, and packages are moved to the tens' bin. The number of counters, or packages as the case may be, is recorded on the blackboard at the rear of the bins. There are few more graphic ways in which the meaning of a numeral such as 35 may be illustrated.



It should be apparent how this device may be used to aid in the understanding of numeration systems to other bases. It is only necessary to change the rules to require five sticks to a bundle, five bundles to a super-bundle, and etc. Both the addition and subtraction algorithms developed in this section may be effectively demonstrated on this device.

The method of checking suggested in the seventeenth paragraph is particularly useful. The check may be shown diagrammatically as follows.



The method gives the student an internal check on his own work, and at the same time involves translation both to and from another base along with adding in both bases. The check may be used with any arithmetic question in another base, and not just in adding and subtracting. It should form an important part of all subsequent work with other numeration systems.

BASE SEVEN NUMERALS (Pages 118 to 123)

Base seven numeration is introduced for a reason other than that of variety. This change to another numeration system involves the student in repetition of all his thinking in yet another system. He will naturally seek to bring to bear upon this material any ideas he can. But there is only one set of ideas that he *can* bring with him from his study of the base five system and that is an understanding of how *any* place value system operates. That such ideas are in fact carried into this new system may be verified by noting how much more quickly students become fluent with this new system.

The text's discussion of the base seven numeration system is parallel to but shorter than the development of the base five system. The place-value box used earlier may be used again here, but it will be found that it is not required by many students.

There are a few side-trips involving other bases which some students or classes might enjoy and find profitable.

Multiplication is properly not dealt with extensively in the text. Most multiplication depends upon quite complex algorithms. It serves no useful purpose to transfer them to another base. Yet there is one idea just beyond what is done in the text which is both interesting and instructive. The multiplication table given near the bottom of page 125 may be extended to include 10_5. A quite astonishing parallel with base ten multiplication will be found. In particular, it will be found that $10_5 \times 10_5 = 100_5$. The significance of this fact should be noted, and students may attempt to see why $10 \times 10 = 100$ in *any* base.

If this is noted, it may then be seen that the advantage of our system over Roman Numerals is not at all that we count by tens, but that we have a place value system. We could change the base of our system and still keep all of the advantages of our system of notation.

Some students might explore bases four, three, and particularly, two. In the base two system, which is examined in detail in **Contemporary Mathematics II**, only two digits are required, 0 and 1. Bases larger than ten require that single symbols be invented for quantities between nine and the base. For example, the first numerals to base twelve might look like 1, 2, 3, 4, 5, 6, 7, 8, 9, X, E, 10.

The symbols X and E are used to represent the quantities 10_{10} and 11_{10}. 10_{12} represents the quantity we know as 12. This base twelve system is known as the duodecimal system. There was a time when a vociferous duodecimal society campaigned for the general adoption of the base twelve system: a class might enjoy listing all things that would have to be changed if we were to change to such a notational system.

Our ordinary systems of weights and measures may be thought of as radical numeration systems in which the base changes from place to place. For example, gallons, quarts and pints may be represented by a notational system in which base two is used from ones to "twos" and base four is used from "twos" to "eights".

An interested student can find accounts of early notational systems in most encyclopedias. The Babylonian system, with at least a partial base of 60, and the Mayan base of twenty from ones to "twenties" and eighteens from "twenties" to "three hundred sixties" are often mentioned. It is clear how the Mayan system was adapted to reckoning time. The Babylonian system may have had a similar origin, but it is more likely that 60 was chosen because of its large number of factors.

TEACHER'S NOTES

TEACHER'S NOTES

5

Fractions

THE MEANING OF FRACTIONS

FRACTIONAL NUMBER LINES (Pages 127 to 130)

The fractional numbers are of such importance that they are often called *the* number system of arithmetic. It is not supposed that students have not met them before, and the authors have been careful not to give the impression of teaching again material which has long since lost its interest. At the same time, the importance of this material makes it imperative that it be thoroughly reviewed. For this reason the elementary properties of the fractional numbers are studied here, but from a considerably more sophisticated viewpoint than ever before. All of the ideas developed so far are brought to bear upon them.

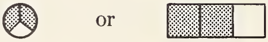
There are a number of important ideas in the first two sections of this chapter. Most of them need not be explicitly identified at this grade level, but the teacher must be aware of them.

a When we work with any abstract mathematical system, we establish the properties of the system according to its postulates. It is possible in this way to play abstract mathematical games which have no connection at all with the real world. But naturally, the systems studied in the school are studied because they are of practical use. The whole numbers and the fractional numbers are exceedingly useful because of the large number of different applications we have for them. We commonly do not think about the variety of different ways in which we apply the whole numbers, because we are so accustomed to looking at the one similarity between these. Students in a classroom, apples in a box, molecules of gas in a closed space, and automobiles on a highway are all freely treated with the whole numbers.

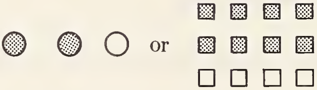
The fractional numbers are used in even more diverse ways than are the whole numbers. Since it is possible to teach children to operate with fractional numbers by giving them only one meaning of such numbers, many students who come to the seventh grade may have a very weak notion of the possible applications of this system. Even some otherwise good modern elementary series are deficient in this matter.

It is a worthwhile exercise to have a class suggest all of the different situations in which we may use a fraction such as $\frac{2}{3}$. The following four will almost always be suggested;

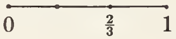
(i) Part of a whole



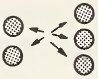
(ii) Part of a group



(iii) A measure



(iv) A rate pair



These situations, of course, bear some resemblance, otherwise we could not use the same fraction, $\frac{2}{3}$, to describe them, but if one approaches these situations naively, one sees that they are really quite dissimilar. The first involves cutting one object up. The second involves partitioning a group. The third does not involve any cutting or partitioning, $\frac{2}{3}$ is simply a label for a point on a ruler, and the fourth uses the fraction $\frac{2}{3}$ to identify the relationship between two different groups.

A fifth and very important meaning of a fraction is often not encountered before the seventh grade, and that is that the fraction $\frac{2}{3}$ may represent an indicated division. That is, $\frac{2}{3}$ may be interpreted as $2 \div 3$.

Students in the seventh grade often do not have much difficulty in coming to terms with any one of these meanings separately, but sometimes do have trouble reconciling two of them.

In **Contemporary Mathematics,** it has been felt best to continue to use the meanings of a fraction intuitively at this level, using only those meaning that are needed in context. **Contemporary Mathematics II** will consciously examine the possible interpretations of a fraction. For this reason, only the two interpretations needed immediately are mentioned in the first section of this chapter. $\frac{3}{4}$ is shown as a cut, and then the new interpretation, indicated division, is introduced. This interpretation is reconciled first with the cut interpretation and then with the measure interpretation.

Without any formal discussion, the teacher should look for the understanding of the idea that if we think of $\frac{3}{4}$ as three of four identical parts of one object, or if we think of it as the result of dividing three objects into four identical parts, we are thinking of the same thing. In the same way, if we think of $\frac{3}{4}$ as the point between 0 and 1 on a number line to the left of which there are three parts equal in length to the part to the right of it, we are thinking of the same point as if we think of a length one quarter as long as the distance from 0 to 3.

b These sections also extend the important distinction between a number and its names to the study of fractional numbers. In traditional language, both the numbers and the symbols have been called 'fractions'. The more sophisticated discussion which follows in this chapter makes it imperative that if the distinction has not been made before, it be made now.

The number is now called a *fractional number* and the symbol for a fractional number is called a *fraction*. The former term is somewhat cumbersome for ordinary discourse, and may be replaced with the simple word *number* for most purposes. The teacher should make every effort to preserve this distinction in his own speech and, to a lesser degree, in the classroom.

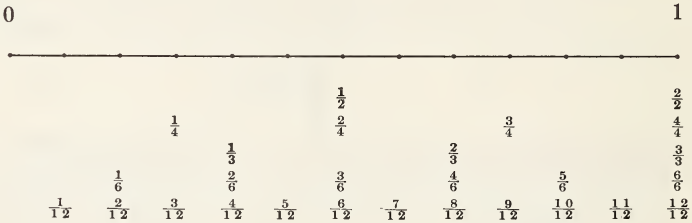
As with the whole numbers, a number of exercises are useful here. The teacher should note the following properties of the two entities.

$\frac{2}{5}$	'$\frac{2}{5}$'
(i) does not have a numerator	(i) has a numerator
(ii) does not have a denominator	(ii) has a denominator
(iii) does not have a bar across the middle	(iii) has a bar across the middle
(iv) equals $\frac{4}{10}$	(iv) does not equal '$\frac{4}{10}$'
(v) is greater than $\frac{1}{5}$ and less than $\frac{3}{5}$	(v) is the same size as '$\frac{1}{5}$' and '$\frac{3}{5}$'
(vi) is the answer when 2 is divided by 5	(vi) may be interpreted as an instruction to divide 2 by 5

In most situations it is quite clear whether we are using a fractional number or a fraction, but the teacher should note that there are a few situations in which the distinction is not clear. Since the distinction is intended to aid the student's understanding, rather than to confuse him, it is best not to analyze these mixed situations. The most important is noted in the last point in the second column above. Some of the interpretations of fractional numbers depend upon the fact that a fraction has a numerator and a denominator. When a person thinks of dividing 2 pies among 5 persons, he may well write $\frac{2}{5}$ while thinking, $2 \div 5$. He is using the *fraction* to indicate division. He may then cut each pie into five parts, giving each person two parts. Again, he is mainly using the *fraction* but as the result of some intuition about fractional numbers, as an instruction as to what to do. It is possible to trace through these situations, deciding when shifts have been made from fraction to fractional number and back again, but the discussion is quite pointless in a seventh grade class.

c One of the important reasons for noting the difference between fractions and fractional numbers lies in our algorithms for calculation with fractional numbers. For a given whole number, say six, there is one 'best' numeral, 6. It is rarely more convenient at this grade level to name six as $5+1$, or in some other form. Many students will persist in thinking that the symbol '6' really *is* six and that symbols such as '$5+1$' are not altogether honest.

This matter of working with different names is the main new skill in working with fractional numbers. *It is essential that all students recognize that while $\frac{1}{2}$ and $\frac{2}{4}$ are different fractions, they name the same fractional number.* In **Patterns in Arithmetic** the number line is used extensively to help students to this understanding, and it is repeated only briefly here on page 130. On a number line such as is shown on the following diagram it is readily seen that several different fractions name the same point.



Students who are not already acquainted with such diagrams should study this one and others like it closely. It is a good plan to verbalize as follows: "Since '$\frac{2}{3}$' and '$\frac{8}{12}$' name the same point, they must be different names for the same number". The teacher will note that a point on a line is not a number, it too is a way of naming a number, but a number line is such an excellent model for the fractional number system that this need not be pursued in a seventh grade class. The point may be identified with the number.

The language from earlier grades, that '$\frac{1}{2}$' and '$\frac{2}{4}$' are different names for the same number, is first repeated, then the relationship between these fractions, that they are *equivalent* fractions, is stated in more sophisticated form. It is not necessary to note any formal relationship between equivalent fractions at this point. A later section of the chapter deals with this matter fully.

d When fractions are defined, the most natural definition is sought. Formally, fractions are defined as ordered pairs of whole numbers with the second number not zero, and fractional numbers are then defined as equivalent classes of fractions. At this level it is adequate to say that the numerator may be a whole number and the denominator a natural number. With more able classes, the teacher may wish to note that this involves mixing different systems and that the situation is a little tidier if we use whole numbers in both the numerator and denominator but insist that the denominator may not be 0.

A FIRST LOOK AT MULTIPLICATION OF FRACTIONS (Pages 130 to 134)

It has already been noted that this chapter does not intend to develop arithmetic with fractional numbers in the way in which it was done in earlier grades. The attempt here is to help the student to view the system from a more sophisticated point of view.

This section uses the intuitive notion of equivalence of fractions suggested in the first section, particularly with regard to fraction names for whole numbers, to make the procedure for multiplying fractional numbers seem reasonable. While this treatment does not expose the full mathematics of the situation, it is honest.

Mathematically, multiplication of fractional numbers could be defined in any way at all. But the system would not be very useful for ordinary purpose if the products of fractional numbers and of their equivalent whole numbers did not turn out to be the same. The development in the text leads the student to note that the procedure for multiplying fractional numbers is at least consistent with the multiplication of whole numbers.

EQUIVALENT FRACTIONS (Pages 135 and 136)

In earlier grades, students learned that they could find a fraction equivalent to a given fraction by multiplying the numerator and denominator of the given fraction by the same number, or by dividing the numerator and denominator of the given fraction by the same number. In **Patterns in Arithmetic** this procedure was justified by appeal to as many different models as possible. In this section the procedure is justified on new grounds. It is seen that any fraction may be renamed if the fractional number it represents is multiplied by 1, using some fraction name for 1. This is a most important new idea. Considerable further work, both in this text and in further years will be based on it. In **Contemporary Mathematics II**, the idea is made yet more general as the *Principle of Equivalent Fractions*.

FRACTIONS IN THEIR LOWEST TERMS (Pages 136 and 137)

In this section, it is seen how the principle enunciated in the last section may be extended to the renaming of fractions as fractions in lowest terms.

COMPARING FRACTIONS (Pages 137 to 140)

This section shows how fractional numbers may be ordered if fractions for them are plotted on a number line. Students who are familiar with this idea from previous years may omit this part of this section. Students who are not familiar with this idea should examine it thoroughly, because this model will be used to introduce the idea of the density of the fractional numbers later in the chapter.

This section also studies the renaming of fractions representing fractional numbers larger than one from first principles. There will be students for whom this means of renaming mixed numerals is quite new and startling.

COMMON DENOMINATORS (Pages 140 and 141)

Students have, of course, found common denominators before, but it is now possible to restate the procedure using the language of the chapter on factors and primes. The examples used in the text are quite simple in comparison with the problems given in the earlier chapter on the finding of least common multiples. With average and brighter classes the teacher might well present a few questions after the manner of exercise 4, in which the denominators are more complex. e.g.,

g $\frac{9}{18}, \frac{29}{60}, \frac{23}{45}$ **h** $\frac{32}{40}, \frac{19}{25}, \frac{21}{32}, \frac{39}{50}$

The teacher might also observe that it is not necessary to work with the least common multiple of the denominators of a set of fractions in order to compare them. The arithmetic involves larger numbers but is in some ways simpler when the product of the denominators is used, viz.,

4. **a** $\frac{11}{12}, \frac{15}{16}, \frac{9}{10}$

$$\frac{11}{12}=\frac{11\times16\times10}{12\times16\times10}=\frac{1760}{12\times16\times10}$$

$$\frac{15}{16}=\frac{15\times12\times10}{16\times12\times10}=\frac{1800}{12\times16\times10}$$

$$\frac{9}{10}=\frac{9\times12\times16}{10\times12\times16}=\frac{1728}{10\times12\times16}$$

SIMPLIFYING WITH MULTIPLICATION (Pages 142 and 143)

This section completes the work begun in the section, Fractions in Lowest Terms. It is now seen how all renaming of fractions may be regarded as introducing or removing a factor of 1.

ANOTHER LOOK AT MULTIPLICATION (Pages 143 and 146)

It has already been stated that, formally, the way in which fractional numbers are multiplied is a matter of definition. We may use models for two reasons: to help make some arbitrary mathematical definition seem reasonable: to help make students sense some of the ways in which a particular system may be used. This development of the multiplication of fractions is entirely for the second purpose. Most students already know how to multiply fractions. They know some applications, particularly those in which measures are combined. For example, five sticks each $4\frac{1}{2}$ inches long would, end to end, measure $5\times4\frac{1}{2}$ inches.

Most students are less familiar with the application of fractional numbers to area. This is an important application. The development here, then, is not intended to convince students that we should multiply as we do; they already know that. It is intended to give students an intuitive feeling that multiplication of fractional numbers may be applied in situations which have this structure.

This idea will be used from pages 316 on, in particular.

THE ORDER IN WHICH WE MULTIPLY (Pages 147 and 148)

The first six paragraphs of this section culminate in the statement that multiplication of fractional numbers is commutative and associative. Particular attention should be paid to the formal algebraic development of these principles. These developments are a first introduction to the idea of formal algebraic proof. Average and more able students should see that each of these demonstrations is more powerful than any number of specific illustrations. If this concept of *general* proof is gradually introduced in this fashion, students will be much more ready to work fluently with it in later grades.

PROBLEMS INVOLVING MULTIPLICATION OF FRACTIONS (Pages 148 and 149)

The programme we laid out earlier for gradually leading students to attack all problems by writing open sentences for them should be continued, as far as possible, here. It is useful to write two different open sentences for some problems. For example, in problem 6, it is probably clearer at first to write

$$w = 2\tfrac{1}{5} \times 1\tfrac{2}{3}$$

and $t = 1\tfrac{1}{2} \times 3\tfrac{2}{3}$

than it is to write

$$t = 1\tfrac{1}{2} \times (2\tfrac{1}{5} \times 1\tfrac{2}{3})$$

$$\tfrac{11}{5} \times \tfrac{5}{3} = 3\tfrac{2}{3}$$

The latter form often involves simpler arithmetic and provides one motive for learning to write entire problems of this sort as one equation.

RECIPROCALS

DIVISION OF FRACTIONS (Pages 149 to 153)

There are several good ways to introduce the division of fractions. When they are studied for the first time, as in **Patterns in Arithmetic 6,** this topic is introduced from as many different points of view as possible. Here the route through the reciprocal is formally reviewed. The statement that division and multiplication are inverse operations might well be reviewed before beginning this section.

PROBLEMS INVOLVING DIVISION OF FRACTIONS (Pages 153 and 154)

As before, students should be encouraged to write open sentences for these problems.

ADDITION OF FRACTIONS (Pages 154 and 157)

It will be noted that in this chapter, addition and multiplication of fractional numbers are not in their traditional order. The reason should be apparent by now. This chapter is concerned with examining the fractional numbers from principle, rather than as a system of arithmetic. The fundamental new idea is the idea of equivalent fractions, which is directly related to the multiplication of fractional numbers.

This section should be seen as having three main objectives.

a The algebraic statement of the algorithm for addition in paragraph 7. All students should study this demonstration closely. The teacher will note that the formulas finally established may be re-applied to the addition of fractional numbers, given a more compact algorithm than the student has previously had. The examples in the paragraphs from the fourteenth to the end of the section may be used for practice in using the addition algorithm in this new form. viz.;

$$\text{since } \frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{bd}$$

$$\frac{3}{4}+\frac{2}{3}=\frac{(3\times 3)+(4\times 2)}{3\times 4}=\frac{9+8}{12}=\frac{17}{12}=1\frac{5}{12}$$

With some practice, much of the writing in this algorithm may be omitted and many sums written after inspection. When three or more fractional numbers are to be added, it will again be noted that the product of the denominators is to be preferred to their lowest common denominator. viz.;

$$\frac{2}{3}+\frac{3}{4}+\frac{1}{5}=\frac{(2\times 4\times 5)+(3\times 3\times 5)+(3\times 4\times 1)}{3\times 4\times 5}=\frac{40+45+12}{60}$$

$$=\frac{97}{60}=1\frac{37}{60}$$

The pattern used in writing numerators may soon be 'felt in the fingers'.

b Addition of fractional numbers is commutative. In this case the principle is stated algebraically but not proven. Classes of more able students or such students in other classes might well repeat the development of paragraph 7 for $\frac{c}{d}+\frac{a}{b}$ and then, comparing the two results, prove that they must be the same fractional number. Reasons must be given for each step.

$$\frac{c}{d}+\frac{a}{b}=\frac{c}{d}\times\frac{b}{b}+\frac{a}{b}\times\frac{d}{d}=\frac{c\times b}{d\times b}+\frac{a\times d}{b\times d}=\frac{c\times b}{d\times b}+\frac{a\times d}{d\times b}=\frac{(c\times b)+(a\times d)}{d\times b}=\frac{cb+ad}{db}$$

Compare $\frac{a}{b}+\frac{c}{d}=\frac{ad+cb}{bd}$ from the text with this development.

$$\frac{ad+cb}{bd}=\frac{cb+ad}{bd}=\frac{cb+ad}{db}.$$

Hence *addition of fractional numbers must be commutative.* This is too difficult for the majority of seventh grade students and has been omitted from the text, but more able students will find it most satisfying and profitable.

c Addition of fractional numbers is associative. As with the commutative principle, the result is simply asserted in the text as the consequence of several trials. More able students might attempt the algebraic demonstration, particularly if they have been guided through the demonstration with the commutative principle.

PROBLEMS INVOLVING ADDITION OF FRACTIONS (Pages 157 and 158)

As before, open sentences should be written.

MULTIPLICATION AND ADDITION OF FRACTIONS (Pages 158 and 159)

There is one main idea in this section: that *multiplication of fractional numbers obeys the distributive principle.* This is of more than passing interest. The ordinary multiplication algorithm is quite inefficient with many question, such as $32\frac{1}{2}\times 16$, yet questions of this sort are of quite frequent practical occurrence. The distributive principle may be used to simplify calculation considerably in these cases. The teacher is advised to do considerable work on this idea with slower students who are likely to enter vocational programmes in the near future. This is, for them, a most practical application of an underlying principle. While these questions should first of all be written in this form;

$$8\times 12\frac{1}{2}=8\times(12+\frac{1}{2})=96+4=100,$$

later, this form should be used:

$$\begin{array}{r} 18\frac{1}{3} \\ \times\ \ 6 \\ \hline 2 \\ 108 \\ \hline 110 \end{array}$$

With some students, who are probably not in the group for whom this exercise is recommended, it may be possible to work such questions in double distributive form.

$$\begin{array}{r} 12\frac{1}{2} \\ \times 4\frac{1}{3} \\ \hline 4\frac{1}{6} \\ 2 \\ 48 \\ \hline 54\frac{1}{6} \end{array}$$

Notice that $\frac{1}{3}\times\frac{1}{2}$ and $4\times\frac{1}{2}$ have been combined in one partial product.

This procedure is not of any great value, because most practical applications involving numbers of this sort will probably be solved using decimals.

The teacher may have thought of assigning the proof that the distributive principle holds in the fractional numbers to more able students. Unfortunately, this demonstration involves algebraic manipulation beyond the grasp of students at this level. A few very able students may attempt it.

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{cf+de}{df}=\frac{a\times(cf+de)}{bdf}=\frac{acf+ade}{bdf}$$

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}=\frac{ac}{bd}+\frac{ae}{bf}=\frac{acbf+aebd}{bdbf}$$

The first line shows the working when the distributive principle is not used: the second line shows the working when it is. The teacher will note that there is a common factor of b in the numerator and denominator of this expression. It will not appear if the fractions in $\frac{ac}{bd}+\frac{ae}{bf}$ are taken to the least common denominator to be added. It should be repeated that this discussion is well beyond the experience of most seventh grade students.

SUBTRACTION OF FRACTIONS

PROBLEMS INVOLVING SUBTRACTION OF FRACTIONS (Pages 160 to 163)

The teacher will note that the naive approach of earlier grades is replaced by the more formal notion that addition and subtraction are inverse operations. These two views are reconciled in the section. This relationship between addition and subtraction may be used in particular when open sentences have been written for problems. The original open sentence for a problem should always follow the pattern of the problem as closely as possible at this grade level. But once the open sentence has been written, it may be re-written as above.

For example, in problem 5, the equation is

$$27\tfrac{7}{10}-n=24\tfrac{3}{5}$$

but this equation may now be re-written according to the relationship between addition and subtraction once or twice if necessary.

$$27\tfrac{7}{10}=n+24\tfrac{3}{5}$$
$$27\tfrac{7}{10}-24\tfrac{3}{5}=n$$

FRACTIONS BETWEEN FRACTIONS (Pages 163 to 166)

This section and the following one introduce, for the first time for most students, an important property of the fractional numbers, their density. The idea of the density of the fractional numbers will not only be important in the eighth and ninth grades when irrational numbers are first examined and then studied in detail, but will, for the more able students, begin a study which will extend far into analysis and point set topology. The foundation laid here is most important for students of average and above average ability. This section falls into two parts;

a Showing that there is a fractional number between any two different fractional numbers by the averaging method.

b Showing that there is a fractional number between any two different fractional numbers by the renaming method.

Both of these methods should be studied in detail.

Some students will see immediately upon the statement of this fact that between any two fractional numbers there is an infinite set of numbers. The teacher would do best to let this conclusion develop of its own accord. The examples given in the text tend to this conclusion, but do not force it. The graphical demonstration in the following section will help more students come to this conclusion on their own.

GRAPHING FRACTIONS ON A NUMBER LINE (Pages 166 to 171)

The first four paragraphs of this section re-state and extend the earlier notion that the fractional numbers may be graphed. All students should do this carefully, because this is an important intuitive basis for later work with analytic geometry.

From the fifth paragraph, this section deals with the set of all fractional numbers between a given pair of fractional numbers. Where students have not already done so in the previous section, they must here discover that there are an infinite set of fractional numbers between any two fractional numbers. This is a difficult notion, and one which the teacher should be prepared to expand upon. It is useful to work with a number line and talk about very powerful microscopes. In any interval, no matter how small, we may still find an infinite set of fractional numbers.

It is of value to note that we shall always be able to measure any distances using fractional numbers. No matter how accurately scientists might some day measure distances or any other units, we shall always be able to use fractional numbers to talk about these measurements, because in any interval, no matter how small, we still have an infinite set of different fractional numbers.

This section uses a thickened "line" to represent the set of all points representing fractional numbers on some part of the number line. This is quite satisfactory for the seventh grade. Small open discs are used to indicate that the end-points of such segments are not included.

More able students may raise a number of questions here, and since this matter is of such importance later, it is worthwhile discussing some of these points with such students.

For example, it may be noted that there is no greatest fractional number in the set of fractional numbers less than $\frac{1}{2}$. This contradicts our intuition. It is intuitively repugnant that we should be able to imagine ourselves travelling along the number line and suddenly find ourselves among a set of points, without having reached a single first point. The difficulty arises, of course, from the attempt to use intuitive notions such as travelling along a line in connection with such notions as infinite sets of points.

There is another equally astonishing matter which may arise. The density of the fractional numbers, and the fact that a thickened line segment was used to indicate certain sets of them, both tend to have one intuitively accept the proposition that the fractional numbers fill up the line entirely. This is not so. All of the irrational numbers may also be represented by points on a line, and none of these points are named by fractional numbers. This matter will be mentioned in **Contemporary Mathematics II** but will not be studied in any detail until the ninth grade.

PROBLEMS USING FRACTIONS (Pages 171 to 173)

As before, open sentences should be used to demonstrate the structures of these mixed problems. Slower students might be encouraged to use several different open sentences where necessary. More able students might be encouraged to write as many different open sentences for one problem as possible.

TEACHER'S NOTES

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6 Decimal Fractions

DECIMAL NUMERATION AND FRACTIONS (Pages 176 to 180)

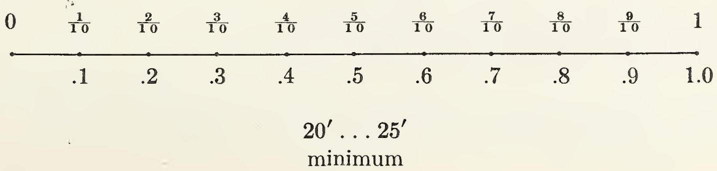
This section re-teaches the idea that decimal numerals are other names for fractional numbers, but does so from a more formal point of view than in the sixth grade. In this section full use is made of the idea of place value, exponents, and the previously stressed idea that all fractional numerals have many names.

The teacher should note that while this section stresses that all decimal numerals are other ways of naming fractions, it is not here asserted that all fractions may be renamed as decimal numerals. Only those examples in which the denominator of the given fraction (in lowest terms) divides some low power of ten are considered. Eventually this statement is true, but proving it involves division with decimal numerals, which has not yet been reviewed.

Less able students should complete many exercises of the sort given in paragraph 17. The understandings established here are most important for later work. More able students might well both work with larger denominators such as 64, 125, 160, etc., and be asked to see why fractions with denominators such as 3 cannot be translated to decimal form in this way. The teacher might suggest that the student decide whether or not 3 divides 9, 99, 999, 9999, etc. If it does, could it divide one *more* than any of these numbers? It is not so easy to demonstrate that denominators such as 7 and 11 cannot be renamed in this way. The discussion of them is best delayed until recurring decimals are studied.

DECIMAL FRACTIONS AND NUMBER LINES (Pages 181 and 182)

Many students have considerable difficulty in ordering decimal numerals. Besides re-emphasizing the relationship between the fractions and the decimal numerals, the plotting of decimal numerals on a number line helps to establish some notion of order among them. If the teacher has time there is great value, particularly for slower students, in preparing a large number line to stretch completely across the front of the room above the chalkboard. On such a number line, thousandths may be represented easily by points and ten-thousandths may be estimated. The relative insignificance of digits to the right of a given digit in determining order is emphasized in a way that is not easily duplicated by any other means.



ADDITION AND SUBTRACTION WITH DECIMAL NUMERALS

PROBLEMS USING ADDITION AND SUBTRACTION OF FRACTIONS (Pages 183 to 185)

If a student is instructed to keep decimal points under decimal points and proceed as before, he has no difficulty finding answers to addition questions involving decimals. But the work he does in this way cannot be expected to help him understand decimal notation, for apart from giving some practice in making dots it is simply practice in adding and subtracting. This section presents, as briefly as possible, the rationale for adding and subtracting when fractional numbers are named in decimal form. The teacher should notice that as a result of this discussion it is more relevant to speak of aligning the ones' figures.

MULTIPLICATION USING DECIMAL FRACTIONS (Pages 185 to 190)

Three means of placing the decimal point when two numbers expressed in decimal form are multiplied are examined:

a The analogy with fractional numbers. This is certainly the most meaningful of the three approaches. **Patterns in Arithmetic 6** and other texts use this approach thoroughly in the sixth grade, and some students may find this development to be merely review. Other students may need to study this development closely. Sufficient practice with this procedure leads the student automatically to the second means.

b Counting the number of places in the multiplier and in the multiplicand. This procedure is not to be decried, but should be based on a thorough understanding of why the rule works.

c The third method is not so often encountered before this level. It is quite valuable for students aiming at any technical work involving slide rules and other computing devices and should not be dismissed as a curiosity. This method involves disregarding all decimal points, multiplying, and placing the decimal point in the product by inspection. This procedure is not only useful arithmetically: by encouraging estimation it also promotes an understanding of the structure of the problems in which decimal notation is used.

Multiplication where there are factors of ten and tenths is introduced in this section. Some students may make discoveries about such multiplication. The teacher should note that such multiplication is treated explicitly later in the chapter and should not be pursued in that fashion here.

DIVISION WITH DECIMAL FRACTIONS

SHORTENED METHOD (Pages 190 to 195)

A good presentation of the material of these two sections, with resulting understanding on the part of the students, makes some demands upon the teacher. The following should be noted:

a The first paragraph of this section may be expanded considerably with some classes. This is the first suggestion that fractions may be used as part of the quotient when division questions are presented this way. The teacher may find it useful to write out a few questions this way,

Similarly:

$$\begin{array}{r|l} 7\overline{)103} & 10 \\ \underline{70} & \\ 33 & \\ \underline{28} & 4 \\ 5 & \frac{5}{7} \\ \underline{5} & \\ 0 & 14\frac{5}{7} \end{array}$$

From this point on, it is not necessary to leave remainders when division questions are posed this way. In short form, we may write,

$$\begin{array}{r} 84\frac{3}{8} \\ 8\overline{)675} \\ \underline{64} \\ 35 \\ \underline{32} \\ 3 \\ \underline{3} \\ 0 \end{array}$$

b Exactly the same notion is used when quantities less than one are permitted in the quotient when we divide using decimal numerals. Many students are not aware that division questions involving decimals may be worked as in paragraph four of this section. It may be instructive to have a class work several similar examples this way. It should be seen that the eventual strategy of renaming the divisor and the dividend is one of convenience.

c In the fifth paragraph of this section, it should be noted that any one of several different multipliers will be convenient in simplifying the question. It is most instructive to have students work with $\frac{10}{10}$, $\frac{100}{100}$, $\frac{1000}{1000}$, etc. in several different examples.

In most of the examples on pages 192 and 193, ample space is left to the right of the dividend in the worked out examples. This is because the strategy for such calculation is somewhat different on paper from in the text, and the authors wished to avoid having the worked examples look too convenient. When 125 is divided by 250, for example, it may not be clear how many places will be required. The student may begin with two places. He may discover later that the second place is superfluous, or he may discover that a third place is required. He would then amend the dividend by annexing a zero, and amend each difference as required.

d It is quite sufficient to leave the procedure at this level for now. Recurring decimals and rounding off will be studied presently. Notice that the division algorithm for renaming common fractions in decimal form is introduced here.

e The shortened method is a logical extension of the shortened method as used in the division of whole numbers. It is a good plan to have many exercises worked in both forms.

PROBLEMS USING DECIMALS (Pages 195 to 197)

As before, open sentences should be used with these problems. When they are used consistently with each set of problems throughout the book, the student builds up a large fund of experience on which he can draw when he must concentrate more fully upon the solution of equations.

REPEATING DECIMALS (Pages 198 and 199)

The full form of the division algorithm is most appropriate to the studying of repeating or recurring decimals. It will be seen that not only does the recurring pattern of the figures in the quotient show up, but also the diminishing remainder. The fact that considerable extra writing is required is of no consequence here.

When a student begins to work a question in which he is not sure whether the quotient will terminate or recur, he may leave ample space for more zeros between the dividend and the vertical bar. It is then possible to annex zeros if they are required.

ANOTHER LOOK AT MULTIPLICATION AND DIVISION USING MULTIPLES OF TEN (Pages 200 to 202)

Notice that the resulting rules are first stated in terms of moving digits rather than the decimal point. It is worthwhile exploring this procedure more formally using the distributive principle.

$$\begin{aligned} & 10\times(17.64) \\ &= 10\times(10+7+\tfrac{6}{10}+\tfrac{4}{100}) \\ &= (10\times10)+(10\times7)+(10\times\tfrac{6}{10})+(10\times\tfrac{4}{100}) \\ &= 100+70+6+\tfrac{4}{10} \\ &= 176.4 \end{aligned}$$

It will be seen that each figure has been moved one place.
Multiplication by .1 may be shown in the same way.

SCIENTIFIC NOTATION (Pages 202 to 204)

It is relatively easy for most students to learn to rename in this form. The question most likely to come up is ‘Why bother?’. The teacher may demonstrate, without mentioning any rules, how this notation facilitates calculation in which powers of tens are manipulated with great ease. There is another more subtle reason with which the teacher should be acquainted. Scientific notation obviates a vexing problem in significant figures. When we write that the distance between two posts is 2000 feet, there is no way of showing immediately how many of these figures are significant. It is conceivable that only the '2' is significant, but how would the measurer show that his measurement was correct to the nearest hundred feet? He would wish to make the first 0 significant, but the others not. He can easily do so with scientific notation. The symbols 2×10^3, 2.0×10^3, 2.00×10^3, and 2.000×10^3 indicate a calculation correct to the nearest thousand, hundred, ten, and single foot respectively.

More able students may examine the table on page 204 with this in mind. It seems unlikely that the diameter of some planets would be given to the nearest ten miles while the diameters of others would be given to the nearest thousand. Yet there is no way of telling from the table whether the given diameter of Saturn is to be regarded as 7.5×10^4 miles, or 7.50×10^4 miles or 7.500×10^4 miles.

ROUNDING OFF DECIMALS (Pages 204 to 209)

Teachers are occasionally astonished at how weak some students seem to be at this comparatively simple task. Inability to perform this task may usually be traced to early lack of understanding of place value. The larger number line suggested at the time is particularly useful here.

The symbol $\simeq$ should be used rigorously. A student should never be permitted to write $=$ unless the quantities named are strictly equal.

Paragraphs eight and nine of this section should be studied closely and expanded with some classes. They indicate how rounding off may be used to preserve a reasonable number of significant figures in multiplication questions, but more important, indicate how rounding off may be used to obtain practical answers in the division of decimals.

The full treatment of this topic appears in **Contemporary Mathematics II** where a systematic and efficient algorithm for practical division with decimal notation is presented. It is more important at this level that students develop a strong understanding of the decimal notation. Teachers who feel impatient at this delay should reflect that few, if any, students now leave school after the seventh grade. The diverse programmes studied at the secondary level by many students have made it necessary that basic understandings as well as rote computational skills be developed. When the material is organized in this way, by the end of the eighth grade students have mastered both an efficient division algorithm and a sufficiently large body of related understandings to be able to apply it intelligently.

There is no reason why some more able classes should not complete their knowledge of the division algorithm in the seventh grade, but it must not subtract from time devoted to the development of the key ideas of ratio and geometry in the following chapters.

The most efficient division algorithm for students who have learned the division algorithm subtractively differs from the traditional algorithm only in that the decision as to how many places the division is to be taken is made *before* calculation begins. For example, to find 293 ÷ 11, with the calculation carried to hundredths, the pupil might write;

$$\begin{array}{r|r}
11)293.00 & 20.00 \\
\underline{220.00} & \\
73.00 & 6.00 \\
\underline{66.00} & \\
7.00 & .60 \\
\underline{6.60} & \\
.40 & .03 \\
\underline{.33} & \\
.07 & 26.63
\end{array}$$

It would then be possible to write the quotient correct to the nearest tenth. If a quotient were required to the nearest hundredth, the calculation would be carried to thousandths. After some experience with this form, both the teacher and the student will note that all of the decimal points are inserted as a matter of course. It is sometimes accepted as a convention that they will be omitted, except in the dividend and the quotient, as follows;

$$\begin{array}{r|r}
11)293.00 & 2000 \\
\underline{220\ 00} & \\
73\ 00 & 600 \\
\underline{66\ 00} & \\
7\ 00 & 60 \\
\underline{6\ 60} & \\
40 & 3 \\
\underline{33} & \\
7 & 26.63
\end{array}$$

The algorithm in this form is developed in detail in **Contemporary Mathematics II**.

TEACHER'S NOTES

TEACHER'S NOTES

7

Ratio and Proportion

While students work more or less intuitively with the idea of ratio from early in the primary grades, a direct study of ratio with reasonably complete algorithms for calculation involves a mastery of the properties of the fractional numbers beyond the sophistication of most students prior to the seventh grade. This will be the first time these students will have explicitly studied ratio and studied the way in which it may be applied to the solution of many problems in the real world. Therefore, this is a particularly practical unit.

Occasionally those who examine superficially units such as this in modern arithmetic texts complain that they do not find 'practical' applications. This lack is more apparent than real. The main change has been from a topical classification of problems to a *structural* classification. If one is not concerned with the mathematics of the situation, one may have units on 'transportation mathematics', 'shop mathematics', 'business mathematics' etc. In a table of contents, such headings look 'practical'. In fact, such an organization of topics is most inefficient. The basic mathematical structures behind such categories are what determine a student's understanding, not the accidental vocabulary in which the problem happens to be clothed.

The more careful reader will find ample problems of the kinds that have always been considered important throughout the text, associated with the study of the appropriate mathematics. This is particularly evident in this chapter. An understanding of the mathematics of ratio is fundamental to many different applications; more in fact than can possibly be treated in a chapter of this size. The teacher will find that the class that has mastered the principles of this chapter will be able to solve the problems contained in it and many will, for themselves, see this same structure in problems not previously discussed. This is the great strength in having children learn to apply their knowledge of the structure of mathematics rather than learn specific solutions to specific kinds of problems.

RATES AND EQUIVALENT ORDERED PAIRS (Pages 213 to 218)

The relationship between rates and ratios is a close one. Ordinarily, the word 'rate' is used when there are units attached to numerals. For example, a person might say that he is travelling at a rate of 45 miles per hour. This statement really conceals the numbers 45 and 1, and the units miles and hours.

A ratio is more commonly thought of as an ordered pair of numbers such as 4:3. We calculate with these ratios according to fixed rules, just as we add, subtract, multiply, and divide whole numbers.

It will be seen that rates are quite concrete things, closely attached to experience, while ratios are more abstract. But since rates are commonly translated into ratios for purposes of calculation and ratios are applied in situations which involve rates, there is always some overlap in usage between the two words and many situations in which, by minor changes in vocabulary, either term might be used. Except where common usage decrees otherwise, which is sometimes the case, this chapter preserves the distinction between ratio and rates mentioned above.

The distinction between these words is not as important as the teacher's understanding of the programme which leads the student from his naive notions of rate to the abstract idea called ratio, and from a firm grasp of the abstract idea of ratio to many more applications than the student has previously imagined.

Common usage dictates that rates will usually be spoken of in the form 'miles per hour', '$3 per person', and so on. There is not the same universal agreement as to how ratios shall be symbolized. Consistent with the policy established in other texts of this series, which is not to confuse verbal fussiness with mathematical precision, this text introduces the three main ways in which ratio is commonly symbolized. The merits and shortcomings of each notation are discussed in the *Teacher's Guide* rather than in the text.

After the primitive idea of rate is introduced, the first section indicates how the pair of numbers involved in a rate may be abstracted and studied by themselves. For the purpose the general mathematical notation for ordered pairs is used. The teacher should realize that this notation, of the form (7, 3), is used quite generally in mathematics where any ordered pairs are to be discussed. If a particular kind of ordered pair is to be studied at length, usually some more characteristic notation is adopted. For example, when the fractional numbers are first studied in mathematics, they are symbolized in the form (2, 3); this collection of symbols would ordinarily represent $\frac{2}{3}$. This latter notation is later agreed upon to identify this particular kind of ordered pair. When the co-ordinates of points are first studied in co-ordinate geometry, *they* are usually introduced in the form (12, 14). No more particular symbols have ever been adopted in this case.

The (3, 5) form is used here for two reasons; it is sometimes used in contemporary texts to indicate ratio so that students might well be acquainted with it, and also it introduces the general mathematical way of denoting ordered pairs.

This general notation and the common practice of denoting ratios by use of fractions are introduced at the same time. Ratios may of course be renamed in exactly the way fractions are renamed, hence writing ratios in fraction form is very suggestive. The teacher should often accept one notation for a ratio and immediately ask that it be presented in the other form.

The final paragraphs of the section are devoted to assisting the pupil become familiar with renaming ratios, both when they are denoted as ordered pairs and as fractions.

RATIO (Pages 219 to 222)

The third common notation, (9:2) is introduced at the beginning of this section. The emphasis through this part of the section is on facility of translations from one notation to another. It should be particularly noted that when a ratio is denoted by a fraction, either term of the ratio may be the numerator.

Occasionally a student will protest the exercise in paragraph ten. If we may write (45:1) representing a rate of 45 miles per hour, why cannot we simply write (1:1) to represent the rate, 1 pint to one gallon? The brighter student who asks this question may be ready for the answer, which is more involved than may appear to be the case at first sight. If one is mixing some variety of paint, a rate of 1 pint of a first component to 1 gallon of another might well be used, and the person using the rate might well agree with himself to write the ratio 1:1. The advantage of measuring both quantities in the same units is that the resulting ratio is *free of units.* When it is understood that the same units are to be used to measure both components, a statement that the ratio of components is to be 1:8 leaves the user free to measure the components in teaspoonfuls, cups, or barrels; his proportions will still be correct. A few examples will soon convince the brighter student that it is only when the same units are used for both measures that this desirable result is obtained.

The main advantage of the three part ratio notation introduced in paragraph 11 is in economy of writing. Such relationships are otherwise cumbersome to represent. Most often, the relationship between two particular components of the ratio is read from the entire relationship and used to solve some problem.

The brief exercise number 15 is quite important and might well be expanded if it is found that students have difficulty with it. Such complex ratios will later be stated in the form $\frac{3\frac{1}{2}}{100}$ or $3\frac{1}{2}\%$ and will have to be simplified in a routine manner.

PROBLEMS OF RATES AND RATIOS (Pages 222 to 226)

This section, as the name implies, is designed to give some reasonably simple practice in finding ratios in problems. Often the language of the problems suggests rates of the ordinary kind. Ratios are abstracted from these situations and used to solve the problems. Particular attention should be directed to the form of analysis in the model problems. The stating of two ratios, one of them usually involving an unknown, and the statement that these two ratios must be equivalent, will be the basic tool of problem solving throughout this chapter.

The solving of any of these problems by finding the missing component of a ratio involves an algebraic skill which has not been previously studied. Any equation of the form $\frac{2}{3}=\frac{4}{x}$ or $\frac{5}{x}=\frac{15}{21}$ or $\frac{x}{4}=\frac{21}{12}$ leads immediately by the principle of equivalence of rates to an equation such as $2x=3.4$ or $5.21=x15$.

There are two ways in which such an equation may be analyzed at this stage.

a As in the text, it may be asserted that if $2 \times x = 12$, x must be $\frac{1}{2}$ of 12. Similarly, if $15 \times x = 105$, x must be $\frac{1}{15}$ of 105. This argument, while intuitively sound and therefore useful, will be seen to depend ultimately upon the idea of ratio. We are in fact thinking $\frac{x}{2x}=\frac{u}{12}$ where u is now our unknown and we are back where we began.

b A more valid but less compelling argument is based upon the fact that multiplication and division are inverse operations. If $2x = 12$, an equivalent statement is $x = 12 \div 2$ or $x = \frac{12}{2}$.

Most students should see both cases.
In the ninth grade, the correct algebraic reason, that both sides of an equation may be multiplied by the same number, will be given;

$$\begin{aligned} \text{if } 2x &= 12 \\ \tfrac{1}{2} \times 2x &= \tfrac{1}{2} \times 12 \\ x &= \tfrac{1}{2} \times 12 = 6 \end{aligned}$$

There is no need for this formal treatment of equations at this level.

PER CENT (Pages 226 to 228)

A symbol such as 45% may be interpreted in two different ways, both of them mathematically correct. It is correct to regard 45% as yet another numeral for $\frac{45}{100}$ and .45. While this is correct, and will never lead to errors, per cents are rarely used in that way. For example, while it is correct to write $50\% + \frac{1}{4} = \frac{3}{4}$, it is difficult to think of any situation in which such a statement might arise. Per cents are far more commonly used to denote ratios. For that reason, they have been consistently interpreted as ratios in this chapter. This interpretation promotes ability in solving problems in that it obviates the necessity for developing the considerable machinery for solving per cent problems, which has sometimes been the bane of seventh grade arithmetic. At the same time it integrates all work with percentages under the more useful concept of ratio. The teacher who is not familiar with this approach to percentage should study the text problems carefully. One type of problem deserves particularly close attention.

When it is asserted that there are 80 persons in a group and that 25% of them are men, it is traditional with many teachers to think, $25\% = \frac{1}{4}$, so $\frac{1}{4}$ of $80 = \frac{1}{4} \times 80 = 20$. It should be noted that this method of solution involves directly the artificial translation of a per cent into a common fraction that proves so difficult to manage in less obvious problems. The technique here is to write the two ratios $\frac{25}{100}$ and $\frac{n}{80}$ and assert that these ratios must be equivalent. The solution then follows. If this seems a less direct solution it should be remembered that, once mastered, it is considerably more general in applicability than the traditional form, which soon breaks down to three or more 'types' of problems.

MORE ABOUT PER CENT (Pages 228 to 231)

In this section three new ideas are introduced;

a The idea of 100% as representing the ratio 1:1. Here, as elsewhere, it is better to interpret per cents as ratios than as fractions.

b The idea of per cents greater than 100%.

c Situations in which it is necessary to mention fractional per cents and in which it is useful to round off per cents.

This interpretation makes it natural to introduce per cents such as $12\frac{1}{2}\%$. When it is necessary to re-interpret such a per cent as a ratio, we write $\frac{12\frac{1}{2}}{100}$ and rename the ratio in the usual way, (see ex. 15, p. 222).

FRACTION NUMERALS, DECIMAL NUMERALS, AND PER CENTS (Pages 231 to 233)

Once the idea of a per cent as a ratio has been firmly established, the alternative interpretation is presented. It will be noted, however, that fractions are first re-interpreted as ratios and then as per cents, so that the original interpretation of per cent is maintained wherever possible.

PROBLEMS USING PER CENT (Pages 234 to 236)

These problems cover all of the traditional types and involve the more difficult kinds of per cent notation. The basic problem structure is the same here as in earlier problems.

PER CENTS AND BUSINESS ARITHMETIC (Pages 237 to 244)

The teacher should glance over this section and note the way in which many traditional topics of business arithmetic are treated. It should be noted that the underlying structure of all these problems is the same; it is not necessary to complete a large number of problems of each kind in order to give understanding; in some instances one or two examples may suffice. The only new thing in many of these applications is the vocabulary, and much of that is suggested by the work or comes from the student's general experience.

As suggested earlier in this discussion, the structure examined in this chapter runs through a great number of situations that arise quite naturally in the classroom. It will be found that a number of topics, from science and social studies in particular, lend themselves to an application of ratio. For many students, such applications will be more relevant and intrinsically motivating than further applications in business.

The teacher should note the following comments on the vocabulary of business arithmetic:

a Formally, we ought to discriminate between *taxes* and *rates of taxes*, between *profits* and *rates of profits*, etc. Strictly, we ought not to say that a person made a profit of 25% or offered a discount of 30%. These are rates of profit and discount. On the other hand, custom strongly endorses the loose use of language here. The purpose of a section of this kind is to acquaint students with the usage of the business world, not to attempt to reform the world.

Accordingly, the language is generally used in the formally correct way when it is introduced, but in several problems the loose form is given. The teacher should call the class's attention to the formal distinction, but should also call attention to that fact that the distinction is often ignored in educated speech. For example, most educated speakers will refer to a 5% sales tax rather than a 5% rate of sales tax, even though the latter is formally correct.

b In discussing interest, simple interest only is presented. For that reason, it is felt to be unnecessarily cumbersome in this text to refer to these as *simple* rates of interest. It would be a good plan to point out to classes that this means of paying for rented money is really quite rare in the business world. Without going into extensive calculations, it is possible to discuss the nature of compound rates of interest and the effects of such a rate upon the amount of a loan when the time of the loan significantly exceeds one year.

If the principle involved were once mastered, the teacher might use a compound interest table to compare the amount of a loan of $1 for 25 years at 6% simple interest with the amount of a similar loan at 6% compound interest.

TEACHER'S NOTES

TEACHER'S NOTES

8

Geometry

The seventh grade is not the grade for a rigorous deductive development of Euclidean Geometry. At the same time, it is not the place for a weary repetition of the rectangle, the square, the triangle and the circle.

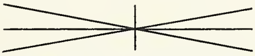
A large body of terminology and concepts are fundamental to any later formal discussion of geometry. This vocabulary and these concepts, often intrinsically interesting in themselves, should form a major part of a course at this level.

The teacher should note that there has been a considerable revision and, often, a tightening of geometric notation in recent years. Probably some of these changes have been unnecessary, but many of them have been useful for the clarification of ideas and all of them are necessary for the student who is later to meet this vocabulary in a more formal context.

POINTS AND LINES (Pages 250 to 258)

This section of the chapter is concerned with developing certain fundamental concepts and words.

In the first place, there is the notion that all geometric figures are to be thought of as collections of points. This, in itself, is different from the once common point of view of a curve as the *locus* of a moving point; the path it traces. It should be noted that that view is now abandoned. *Any geometric figure is thought of as a set of points.* No notion of motion or tracing out is implied. This will be seen to be a more general notion than that of locus if one thinks of the interior of a rectangle. It is difficult to think of this region of a plane as a locus, yet it is quite acceptable to think of it as a collection of points. In the same way, a drawing such as that to the left is difficult to discuss in any clear way as a locus.



The discussion in the text consider a line segment as the result of putting more and more points between two given points. It is intuitively clear to the student that we may extend this set of points beyond the given points to form a line. The segment and the line are both thought of as a set of points. Both of these sets are infinite sets, and the teacher might well encourage students to discover this fact here.

A line segment is defined as being two points together with all of the points between them. Two comments are required here. In more advanced mathematical discourse there is now a practice of defining a line segment as the set of points between two points, excluding the endpoints. At the more advanced level this causes no harm, for one may or may not include the points as one wishes for any particular discussion. At this level the notion of a set of points without endpoints is quite difficult and it is preferable to include the endpoints as part of the segment.

The concept of "betweenness" is used intuitively from the beginning, but the questions at the beginning of the paragraph are intended to help the student clarify his notion of what he means by "between". This concept may be studied more closely in a later grade by students on an academic programme.

There are two ways of introducing the concepts of line and ray. If the notion of line is first established, then we may think of a ray as consisting of a point on the line together with all points on one side of that point.

It is also possible to do as in the text, and first define a ray. We then think of a ray as being a segment, say AB, together with all points such that B lies between each of them and A. A line may then be defined as in the text, using the two rays on a point.

At one time, it was customary to distinguish between *straight* lines and *curved* lines. Current practice is to use the word *line* for what was once a straight line and the word *curve* for what was once any line. This agreement simplifies most discourse. The teacher should observe and consistently use the following notational agreements.

$\overline{AB}$—The segment on A and B.

$\overrightarrow{AB}$—The ray with endpoint A passing through B.

$\overleftrightarrow{AB}$—The line on A and B.

The teacher should note the other vocabulary introduced in this section. The main intent of this section is that the student should develop the minimum geometric vocabulary necessary for concise and precise discussion later. *The emphasis should be on the correct use of words rather than formal memorization of definitions.* There is, for example, no point in having students memorize definitions for *concurrent* and *collinear*; but they most certainly should be able to both pick out concurrent lines and collinear points on diagrams and use this vocabulary in describing relations between points and lines.

PLANES (Pages 259 and 260)

Two important ideas are introduced in this section.

a The notion of a plane as a set of points. These points are introduced in the text whole lines at a time, and this is a useful way to think of the generation of a plane, but the teacher should emphasize that when this process is completed we think of the plane as simply a set of points. With more able classes an interesting analogy with the points on a line may be explored. A *point* divides a *line* into two parts. We now see that a line divides a plane into two parts. At the same time it may be observed that a plane would divide all the points in space into two parts.

b The notion of parallel lines. Through a point, not a line, it is possible to draw one and only one line which does not intersect a given line. These lines are then called parallel lines. The teacher may have read of other geometries in which this is not true and therefore should be cautioned to observe that this statement is strictly true in Euclidean geometry.

In particular, parallel lines do not "meet at infinity". The above notion arose out of some attempts to explain projective geometry (a quite different geometry) in Euclidean terms.

In both this section and the preceding one, a distinction is made between a *line* and a *model of a line*; now a *plane* and a *model of a plane* are distinguished. Lines and planes are geometric ideas, just as numbers are algebraic ideas. We can no more look at a plane than we can look at a number. Just as we use numbers in the real world and attach them to quantities, we use geometry in the real world by looking at *models* of lines and planes. It is a valuable exercise to study these models closely and to see how little they resemble the geometric ideas they are supposed to represent.

For example, a thin model of a line on the blackboard may help us think of a line, but upon close viewing it would be seen to be a loosely connected set of chalk particles. Even if all the spaces between them are filled in, there would be all the spaces between the atoms of chalk. From a different standpoint we see that whatever we draw on the blackboard has some width and some thickness. If anything, what we have on the board is more like a rectangular solid than a line. And if we attempt to imagine the streak of chalk with no width and no thickness, we must ask what it is drawn on; the blackboard certainly is not a plane; and so on. These matters are quite clear to the teacher because we are accustomed to considering the elements of geometry as being abstract entities. We easily forget how divorced these geometric ideas are from the real world. It is necessary to help students realize that there is nothing in the real world anything like a line or a plane. There are only *models* of lines and planes; things which we agree to use to help us think of lines and planes and to which we sometimes later apply our knowledge of geometry.

CURVES (Pages 260 to 262)

Beyond noting the vocabulary of this section, the teacher may wish to pursue the idea of *simple* closed curves. The textual discussion seeks to avoid elaborating an intuitively obvious point, but the teacher may note that a curve could touch itself without crossing itself. A more formal definition is that *it is possible to start at any point of a simple closed curve and trace out the entire curve passing through each point of the curve exactly once.*

ANGLES (Pages 262 and 263)

The idea of what an angle is has also changed in mathematics. At one time it was customary to think of an angle as "an amount of turning". It is quite difficult to give any precise meaning to this notion and it has proved to be much more satisfactory to think of an angle as the union of two different rays with the same endpoint. Later the idea of the *measure* of an angle and the application of such measures to rotations will be introduced; for now the teacher should avoid any reference to turning and consider an angle to be the union of two such rays and no more.

POLYGONS (Pages 263 to 266)

Note that a triangle is a polygon. Some older texts reserved the word 'polygon' for figures with a greater number of sides.

Slower classes might well not be too concerned with such names as 'Heptagon'.

More able classes may be interested in the convention for naming polygons in general. It is now standard practice to abandon these prefixes beyond the octagon and refer to a 9-gon, 17-gon, 46-gon, etc. Notice how this is related to question 9f., page 266.

MEASUREMENT (Pages 267 to 274)

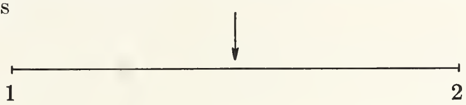
The idea of measurement is quite difficult. Paradoxically, it is generally easiest with slow students, who are not apt to be bothered by some of the difficulties. The teacher is advised to treat slower and more able students quite differently at this point. Both for the measuring of distance and of angles, slower students should mainly become familiar with the units used and the procedures for precise measurement. More able students may explore, often with considerable interest, at least these two problems:

a Once we have selected a unit of length, how do we copy it correctly? May we ever be sure that we have copied it exactly? Since we can never be sure that we have, we must guard our standard of measure carefully. (Recently the standard foot has been redefined in terms of wavelengths of light, obviating the need for a *physical* standard foot).

b Even if we assume that our ruler is an exact copy of the standard ruler, can we ever measure *exactly* with it? Then can we ever find *the* length of an object? If not, then what do we mean by the length of an object? The conclusion of such a discussion might involve averages of some sort, but in any case would include the idea that we must measure as carefully as we can and then agree that by the length of the object we *really* mean the number we announce as the measure of the object.

Formally, measurement of length means assigning to line segments. The vocabulary at the top of page 269 is quite important. The following discussion is intended to help teach that the unit of measure is completely arbitrary; we may select any distance we wish. A student might prepare a report on historical units of measure.

The discussion on page 273 is a traditional introduction to the idea of accuracy. While there is no need to pursue this matter with most classes, the teacher should be aware that this notion, too, is more difficult than it seems. The accuracy with which one may announce the result of a measurement is not really very closely related to the markings on the ruler, even though we often pretend that it is for teaching purposes. As extreme examples, the teacher might consider a one-inch ruler marked in thousandths of inches. If it were used to measure the length of a classroom, only a most naive person would think that his measure was accurate to the nearest thousandth of an inch. At the same time, a person using a yardstick marked only in feet could probably measure the length of a table to about the nearest inch. In fact, our usual discussion assumes this. When we suggest that we can measure to the nearest inch using a ruler marked in inches, we usually draw a diagram like this



A moment's reflection will show that we assume an ability to judge distances much less than one inch with unfailing accuracy when we assume that the user of such a ruler will not err by more than $\frac{1}{2}$ of an inch using such a ruler! If the arrow is almost half way between the given markings, we are in fact assuming an ability to make judgements involving perhaps thousandths of inches on a ruler marked only in inches.

The above discussion is *not* intended for students in general, but the occasional bright student raises these matters, and the teacher should be prepared for them. Eventually, accuracy does not involve the markings on the ruler at all. When a scientist announces the accuracy of a measure, say $12.3 = .2$ inches, he is predicting that anyone who repeats his measuring procedure will come up with a number within .2 of his, and any precise interpretation of that statement involves statistics far beyond the experience of any seventh grade student.

PERIMETERS (Pages 275 and 276)

Notice that $P=2(1+w)$ is not the formula for perimeter: it is a formula giving the perimeter of a very limited set of objects. At the appropriate time that formula will be discussed. At this point the more general idea of perimeter itself is introduced.

THE MEASURE OF AN ANGLE (Pages 277 to 286)

All of the discussion of measurement of length applies with equal force to the measurement of angles. Again it is important that the teacher not speak of angles as *turnings* but simply as the union of two rays. The measure of an angle is again a number assigned to the angle. And, as before, we may select any unit angle we choose. It is an excellent exercise to have a student select his own unit angle, as in the text, and use it many times to measure angles. The labour of marking in all the required points suggests the usefulness of a protractor, and the teacher should guide slower students in particular towards the sudden insight that much of this measuring may be done once and for all on one piece of paper. That piece of paper may then be carried from angle to angle. It has been traditionally noted that such students have difficulty grasping the idea of how a protractor works, and such a development of the idea has been proven to overcome many of these difficulties.

The commonly accepted unit of angle measure, the degree, is introduced in a natural way at the foot of page 279. With more able classes, the teacher may prefer the more formally correct definition which asserts that if a number of rays meet at a point P, the sum of the measures of all the angles at P will be 360 degrees. Even this definition requires some qualification ruling out overlapping angles. It is this complexity of any formally correct definition which commends the use of the less correct but intuitively satisfactory definition of the text.

One of the more important *applications* of angles is in the discussion of rotations. It should be understood that the material on pages 280 and 281 does not constitute a definition of an angle; that has already been done. It introduces the student to the notion that angles may be assigned to rotations and hence rotations may be measured using angle measures.

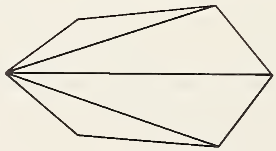
Note the definitions of acute and obtuse angles: it is an interesting exercise to look up the meanings of these words as applied to people.

SOME SPECIAL POLYGONS (Pages 287 to 292)

This section introduces vocabulary as it proves useful to discuss the properties of polygons. *It is better to concentrate upon the properties of the figures rather than the memorizing of definitions.* Any vocabulary which is forgotten or not learned will be treated amply in future years. It is much more important that the student here build up intuitive ideas about triangles and polygons in general. These ideas tend to persist long after formal definitions have been forgotten. This is not to say that correct vocabulary should not be used, simply that *the memorizing of vocabulary should not be an end in itself.*

THE ANGLES OF A TRIANGLE (Pages 293 and 294)

The experimental discovery of the property of the sum of the measure of the angles of a triangle is interesting to most students and may be generalized to situations not mentioned in the text. The student might, using his protractor, measure the angles of a variety of quadrilaterals and examine the sums of the measures. He might come to believe that the sum is always 360°. An examination of a figure such as that at the foot of page 294 would show him why this must be true. The argument may then be generalized to any polygon. After measurement, or even before measurement, a polygon may be divided as follows;



It is then possible to predict the sum of the measures of the angles.

CONSTRUCTING TRIANGLES (Pages 295 to 302)

The teacher who remembers his own high-school geometry will see how, in these constructions, the bases for many later theorems are being laid. At the same time, the student is gaining facility with instruments.

In all of this material there is the opportunity for reference to the earlier discussions of accuracy of measurement. The teacher should see the comment on the following section before teaching this section.

CLASSIFICATION OF TRIANGLES BY SIDES (Pages 302 to 304)

While this vocabulary is gathered together here mainly for future reference, the teacher might well decide not to treat it as a single topic. It would probably be more suitable to introduce it in the context of the preceding constructions. Students might be encouraged to turn to these pages for reference so as to name triangles correctly. *Vocabulary introduced and consistently used in this way is more apt to be remembered than that which is drilled in a formal and, to the student irrelevant, way.*

THE CIRCLE (Pages 305 to 311)

As with curves in general, the circle is studied as a set of points. The teacher who examines the vocabulary of this section in advance will find that it is possible to introduce all of it in the context of the introductory exercises.

There are three matters in this section that should be noted:

a A circle is a set of points. The circle divides the plane into two sets of points, the interior of the circle and the exterior of the circle. No points in the interior or the exterior of the circle are part of the circle. The *circumference* of a circle is a *measure*, not a set of points. At the same time, the word *diameter* is ambiguous. We may speak of *a* diameter of a circle, in which case we are discussing a set of points; a line segment in fact. But we also speak of *the* diameter of a circle. In this case we are referring to the measure of a diameter. This ambiguity in fact causes no harm. It is always clear from the context which one we are referring to. The text uses correct language, but does not stress what might seem a needless distinction to many students.

b π is introduced as the ratio of two measures, the circumference and the diameter. The teacher should note that there is no immediate reason why this ratio should be the same for all circles. The circumferences and diameters of circles vary, why should not this ratio? One of the purposes of the experiments suggested in 6e is to lead to the astonishing fact that this ratio *is* the same for all of the measured objects. Traditionally, many students seem to have acquired the incorrect notion that $\pi = \frac{22}{7}$. This is not only incorrect but interferes with quite important work within two years. The teacher should stress the decimal representation in 7e (without memorization, of course) and that $\frac{22}{7}$, 3.14, or even 3.1416 are all approximations.

c When approximations are discussed, appropriate signs must be used. $\approx$ and $\simeq$ are both commonly used: $=$ is now rarely seen.

TWO-DIMENSIONAL FIGURES (Pages 312 to 320)

The filled-in figures in the text are not designed to add colour to the text. Far too often even able students do not distinguish properly between area and length. Our traditional vocabulary certainly aids the confusion, (square inches, etc.) The teacher must firmly establish that measures of area are not the same as measures of length. The units are different, and do not involve measures of length *at all*. That is why it is most important to follow the discussion of paragraphs 7, 8, 9, 10 closely with the entire class. There is the danger that the class will arrive too quickly at section 12, where the teacher may feel more comfortable.

We could use any shape of object as a unit of area. It may be seen in these paragraphs that there are advantages in using a unit which "fits with itself", such as a rectangular surface, a triangular surface or a square surface rather than a disc. But notice, even here, the temptation to use the words 'rectangle' 'triangle' 'square' and 'circle' none of which could even remotely be measures of area.

The discussion of section 14 is designed to make even more firmly the point that a unit of area is not at all related to units of length. An object of *any* shape could have an area of one square inch.

The remainder of the section attempts to maintain the distinction of area while treating the usual problems of translation of units and measurement.

AREAS OF PARALLELOGRAMS (Pages 321 to 323)

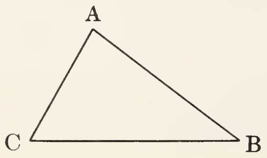
Most students are taken with the cunning of the construction of this section. The teacher will recognize this as preparation for the following discussion of areas of triangular regions, as is the following section.

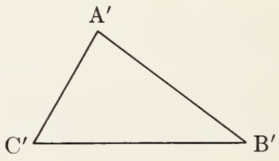
ALTITUDES AND BASES OF TRIANGLES (Pages 323 and 324)

AREAS OF TRIANGLES (Pages 325 to 330)

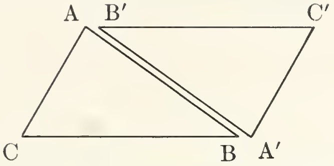
The teacher will notice that in this section the student is led to see how we may find the area of any triangle by partitioning into right triangles. There is ample opportunity in this section to use all of the vocabulary introduced in earlier sections. For example, the teacher might make a point of running through every possible classification of triangle to be sure that this method works.

As a worthwhile side-trip, the teacher might run through the alternative treatment of this topic. For any triangular region, a copy is first made;





The two regions are then fitted together like this;



This parallelogram is then treated as in an earlier section.

UNITS OF SQUARE MEASURE (Page 330)

It is far more important that students begin to understand here that they may calculate with units once a relationship between them is given than that they attempt to memorize all such relationships that they might ever need to know. Most educated adults remember that 1 sq. ft. = 144 sq. in., 1 sq. yd. = 9 sq. ft. and that 640 acres = 1 sq. mi. (1 section). Except in special vocations, where it might be necessary to memorize any number of others, it is difficult to justify the memorization of further equivalents. It is such pointless memorization which quite properly has led some students to despise school arithmetic.

PRACTICE IN FINDING AREAS

PROBLEMS INVOLVING AREA (Pages 331 to 336)

As in the last section, the emphasis should be upon the ability to calculate answers once the relationship between the units given is known, and the translation of physical problems into geometric exercises.

THREE-DIMENSIONAL FIGURES

MEASURING VOLUMES

USING VOLUMES (Pages 336 to 346)

The teacher should note the avoidance of any rapid consideration of the formula for the volume of rectangular solid. The formula is not to be avoided, of course, but since it deals in lengths and can, as with areas, suggest a relationship between volume and length which does not exist, it is important that the notion of volume first be explored carefully.

TEACHER'S NOTES

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TEACHER'S NOTES

9 Graphs

This short chapter is designed to introduce the main kinds of graphs and to develop some awareness of the sort of information graphed on each.

READING AND PREPARING BAR GRAPHS (Pages 347 to 353)

It is stressed that the bar graph is constructed for convenience of comparison. The note in red on page 349 should be discussed. With more able classes, the suggestion of the last sentence might be explored. This would be a good preparation for the more formal examination of means, medians and modes in **Contemporary Mathematics II** and for later work on variation.

The information in the graphs of paragraph 4 is fictitious. That in paragraph 3 and 5b is approximately correct.

Paragraph 6 of this section suggests that graphs may be misleading as well as useful. It is an interesting project to collect examples of both good and misleading graphs for a classroom display. Both kinds are readily available.

READING AND PREPARING LINE GRAPHS (Pages 353 to 355)

The first line graph given presents correct information. It will be noted that throughout this section the emphasis is upon the interpretation of line graphs rather than the drawing of them. The one or two drawing exercises given in each section are sufficient to uncover any significant misunderstandings. They are also a useful exercise in ratio if the teacher should wish to use them for that purpose.

READING AND PREPARING CIRCLE GRAPHS (Pages 355 and 356)

The measures shown on the sectors of this graph are not usually printed on circle graphs. They are given here to emphasize the means by which circle graphs are constructed. As with bar and line graphs, the construction of such a graph is a good exercise in ratio.

TEACHER'S NOTES

10

Review Exercises

While any text is limited in space, a serious attempt has been made to provide sufficient and varied exercise material with each topic. More of such exercises may be required with some classes, depending upon local interests, ability and the teachers' preferences.

These review exercises are designed to be a quick reminder of important earlier ideas. As such, they are most profitably used later in the year. Many teachers prefer to use periods in which it would be difficult to introduce new material for the working and discussion of such review exercises. These periods often come in the spring, when the best case can be made for such review.

While there is a good quantity of exercise material in **Contemporary Mathematics I**, an attempt has also been made to avoid its becoming merely repetitious drill. Classes who have mastered the material of the course will gain more in the long run from an exploration of the auxiliary topics mentioned in the *Teachers' Guide* than from any prolonged drill on mechanical arithmetic.

For that reason, these exercises have been made long enough to cover the key ideas of the year and to encourage discussion of any ideas that may have been forgotten, but they are not prolonged beyond that point.

ANSWERS

CHAPTER 1

PAGE 1

2. (a) {9, 18}   (b) {1, 3, 5}   (c) {6, 8, 10, 12, 14}
(d) {22, 33}   (e) {16, 20, 24}
3. (a) 2   (b) 3   (c) 5   (d) 2   (e) 3

PAGE 2

3. (b) {1, 2, 3, 4, 5, 6}   (c) {0, 1, 2, 3, 4}
(d) {7, 8, 9, 10, 11, 12, 13}   (e) {1, 2, 3 . . . 13, 14}
4. (a) 7   (b) 2   (c) 15   (d) 9   (e) 9
2. (c) >   (d) <   (e) <   (f) >

PAGE 3

(g) >   (h) <   (i) <   (j) >   (k) >   (l) <
3. (b) {0, 1, 2, 3, 4}   (c) {0, 1, 2, 3, 4, 5, 6,}
(d) {0, 1, 2, 3, 4, 5, 6, 7}   (e) {2, 3, 4, 5, 6, 7, 8, 9, 10}
(f) {0, 1, 2, 3, 4, 5, 6}   (g) {0, 1, 2, 3, 4, 5, 6, 7}
(h) {5, 6, 7 . . . 15, 16}   (i) {0, 1, 2}
(j) {0, 1, 2, 3}   (k) {0, 1, 2 . . . 16}
3. Set N = Set M

PAGE 4

(b) Set X = Set Y   (c) Set A ≠ Set B
(d) Set A = Set B   (e) Set A = Set B
(f) Set A ≠ Set B   (g) Set Z = Set F
(h) Set A = Set B   (i) Set K ≠ other set
(j) Set A = Set B
3. The infinite sets are: (a), (b), (d), (e), (f), (i), (j).

PAGE 5

3. The empty sets are: (a), (b), (e), (j).

PAGE 8

2. e.g. (a) 10×5, 49+1, etc.   (b) 13×2 etc.   (c) 2×7 etc.   (d) 1×1, 1+0 etc.

PAGE 9

6. (a) 180   (b) 360   (c) 912   (d) 66
(e) 64   (f) 54   (g) 400   (h) 166

PAGE 10

5. (a) $(4\times10)+(8\times1)$ (b) $(6\times10)+(3\times1)$
(c) $(5\times10\times10)+(0\times10)+(0\times1)$
(d) $(9\times10\times10\times10)+(0\times10\times10)+(0\times10)+(0\times1)$
(e) $(3\times10\times10)+(5\times10)+(0\times1)$ (f) $(7\times10\times10)+(2\times10)+(9\times1)$
(g) $(6\times10\times10\times10)+(1\times10\times10)+(3\times10)+(3\times1)$
(h) $(9\times10\times10\times10)+(0\times10\times10)+(1\times10)+(5\times1)$
(i) $(1\times10\times10\times10\times10)+(0\times10\times10\times10)+(0\times10\times10)+(0\times10)+(0\times1)$
(j) $(2\times10\times10\times10)+(0\times10\times10)+(0\times10)+(1\times1)$
(k) $(3\times10\times10)+(0\times10)+(9\times1)$
(l) $(3\times10\times10\times10\times10\times10)+(0\times10\times10\times10\times10)+(4\times10\times10\times10)$
$+(9\times10\times10)+(0\times10)+(6\times1)$

6. (b) 67 (c) 35 (d) 10 (e) 90 (f) 842
(g) 600 (h) 673,002 (i) 3000 (j) 40,890

7. (b) 8,000 (c) 9,876

PAGE 11

1. (b) i $6+9=15$ ii $7\times6=42$ iii $20\div4=5$ iv $16-5=11$ v $14+12=26$
3. (a) true (b) true (c) false (d) false (e) false (f) true
5. (a) true (b) true (c) false (d) false (e) false (f) false
6. (a) $<$ (b) $=$ (c) $>$ (d) $=$ (e) $<$ (f) $>$
(g) $<$ (h) $=$ (i) $=$ (j) $>$ (k) $<$ (l) $<$
7. a, c, e, f, g, j, k, l.

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10. (a) Numbers become larger (b) $A<B$, $B>A$ (c) No (d) A
12. (a) No <4 (b) No >11

PAGE 16

18. (a) (b) (e) (g) (h)
19. (a) 2 (b) 7 (c) 6 (d) 6 (e) 6 (f) 12 (g) 15 (h) 147

PAGE 17

1. (b) 3, 2, 4

PAGE 18

4. (a) $\{6, 12, 18\}$ (b) $\{21, 28\}$ (c) $\{11, 12, 13, 14\}$ (d) $\{12\}$ (e) $\{1, 2, 3, 4\}$

PAGE 21

5. (a) 18 (b) 20 (c) 51 (d) 1090
(e) 2685 (f) 9906 (g) 4673 (h) 145,705
The sums are the same.

PAGE 22

4. (a) 54 (b) 120 (c) 1104 (d) 30,975 (e) 4212
(f) 62,930
Answers are the same.

PAGE 23

7. (a) 504 (b) 36 (c) 200 (d) 96 (e) 1656
Answers are the same.

PAGE 25

3. (b) $11=6+5$ (c) $32=17+15$ (d) $84=29+55$
(e) $178=70+108$ (f) $487=119+368$ (g) $1015=964+51$
(h) $9000=806+8194$ (i) $75{,}113=6284+68{,}829$ (j) $10{,}000=337+9663$
(k) $25{,}016=428+24{,}588$ (l) $60=21+39$ (m) $325=136+189$
(n) $6{,}000{,}025=26+5{,}999{,}999$

4. (b) $x=11$ (c) $x=7$ (d) $x=19$ (e) $x=8$
(f) $x=15$ (g) $x=285$ (h) $x=60$ (i) $x=519$
(j) 416 (k) 1496 (l) 101

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6. (b) 38 (c) 36 (d) 284 (e) 42 (f) 237 (g) 9910 (h) 921

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4. (b) $x=4$ (c) $x=7$ (d) $x=9$ (e) $x=9$ (f) $x=4$
(g) $x=6$ (h) $x=7$ (i) $x=5$ (j) $x=2$

5. (c) $x=9$ (d) $x=6$ (e) $x=25$ (f) $x=36$ (g) $x=9$
(h) $x=8$ (i) $x=2$ (j) $x=150$

1. (a) 7 (b) 9 (c) 8 (d) 0 (e) 832 (f) 915 (g) 35 (h) 362 (i) x (j) x
In each case, $x+0=x$.

PAGE 28

5. (a) 16 (b) 72

PAGE 29

1. (a) 9 (b) 7 (c) 15 etc.
$a\times 1=a$
The product is the number itself.

PAGE 30

5. (a) $\frac{7}{7}=1$ (b) $\frac{16}{16}=1$ etc.
6. There is no answer.
8. 0 is a whole number.
1. (a) yes (b) yes (c) yes (d) yes (e) yes (f) e.g. 1, 2
(g) yes (h) yes

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9. (a) yes (b) yes; 48 is a multiple of 4 (c) yes Yes it is.

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13. (a) add'n-yes; mult'n-yes (b) add'n-no; mult'n-no
(c) add'n-yes; mult'n-yes (d) add'n-no; mult'n-yes
(e) add'n-yes; mult'n-yes (f) add'n-no; mult'n-yes
(g) add'n-yes; mult'n-yes (h) add'n-no; mult'n-yes
(i) add'n-yes; mult'n-yes (j) add'n-no; mult'n-no

CHAPTER TEST

1. a b f
2. e.g. (a) 5×2 etc. (b) 5×7 etc. (c) 1,000 etc. (d) 125 etc. (e) 9×0 etc. (f) 1
3. (a) 732 (b) 27 (c) 69 (d) 201
4. (a) 8 (b) 40 (c) 5 (d) 3 (e) 10 (f) 26

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5. (a) 7 (b) 14 (c) 7 (d) 115 (e) 216 (f) 205

6. (a) $(4+6)\times 5$ (b) $(12-3)\times 4$ (c) $32+(4\div 2)$
(d) $168-(90-70)$ (e) $46\times(2+6)$ (f) $(23-10)+4+6$
(g) $72\div(4\div 2)$ (h) $(114+6)\times 2$ (i) $24\div(4\div 2)$
(j) $(36+18)\times(2+3)$

7. (a) $(7\times 10\times 10)+(3\times 10)+(6\times 1)$
(b) $(1\times 10\times 10\times 10)+(0\times 10\times 10)+(0\times 10)+(0\times 1)$
(c) $(8\times 10)+(6\times 1)$
(d) $(9\times 10\times 10\times 10)+(0\times 10\times 10)+(1\times 10)+(0\times 1)$
(e) $(1\times 10\times 10\times 10\times 10)+(7\times 10\times 10\times 10)+(0\times 10\times 10)+(6\times 10)+(5\times 1)$
(f) $(1\times 10\times 10\times 10\times 10\times 10\times 10)+(0\times 10\times 10\times 10\times 10\times 10)+$
$(0\times 10\times 10\times 10\times 10)+(0\times 10\times 10\times 10)+(0\times 10\times 10)+(0\times 10)+(0\times 1)$
(g) $(4\times 10\times 10\times 10)+(0\times 10\times 10)+(0\times 10)+(4\times 1)$
(h) $(2\times 10\times 10\times 10)+(2\times 10\times 10)+(2\times 10)+(2\times 1)$
(i) $(1\times 10\times 10\times 10)+(4\times 10\times 10)+(0\times 10)+(1\times 1)$
(j) (1×1)
(k) $(4\times 10\times 10\times 10\times 10)+(6\times 10\times 10\times 10)+(8\times 10\times 10)+(0\times 10)+(8\times 1)$
(l) $(5\times 10\times 10\times 10)+(5\times 10\times 10)+(0\times 10)+(0\times 1)$

8. (a) 400 (b) 3003 (c) 8040 (d) 10,000 (e) 703,201

9. (a) $(2\times 10\times 10)+(9\times 10)+(4\times 1)$ (b) $(4\times 10\times 10)+(8\times 10)+(3\times 1)$
(c) $(4\times 10\times 10)+(7\times 10)+(0\times 1)$
(d) $(2\times 10\times 10\times 10)+(0\times 10\times 10)+(0\times 10)+(0\times 1)$
(e) $(1\times 10\times 10\times 10\times 10\times 10\times 10)+(0\times 10\times 10\times 10\times 10\times 10)+$
$(0\times 10\times 10\times 10\times 10)+(0\times 10\times 10\times 10)+(1\times 10\times 10)$
(f) $(9\times 10\times 10\times 10)+(9\times 10\times 10)+(7\times 10)+(5\times 1)$
(g) $(1\times 10\times 10\times 10)+(4\times 10\times 10)+(8\times 10)+(7\times 1)$
(h) $(1\times 10\times 10\times 10\times 10)+(2\times 10\times 10\times 10)+(3\times 10\times 10)+(4\times 10)+(5\times 1)$

10. (b) $3\times 3\times 3\times 3=81$ (c) $5\times 5\times 5\times 5\times 5=3125$
(d) $12\times 12=144$ (e) $2\times 2\times 2\times 2\times 2=32$
(f) $7\times 7\times 7=343$ (g) $9\times 9\times 9\times 9=6561$
(h) $10\times 10\times 10\times 10=10{,}000$ (i) $13\times 13=169$

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11. (b) $18=11+7$ (c) $368=173+195$
(d) $1{,}000=500+500$ (e) $701=403+298$
(f) $16{,}895=16{,}712+183$ (g) $58{,}064=28{,}889+29{,}175$

12. (c) $n=8$ (d) $n=9$ (e) $n=50$ (f) $n=12$ (g) $n=63$ (h) $n=336$

13. (a) yes (b) yes (c) yes

14. (a) add'n-no; mult'n-yes (b) add'n-yes; mult'n-yes
(c) add'n-yes; mult'n-yes (d) add'n-yes; mult'n-yes
(e) add'n-yes; mult'n-yes (f) add'n-no; mult'n-no
(g) add'n-no; mult'n-yes

15. (a) $\{35, 42, 49, 56, 63\}$ (b) $\{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$
(c) $\{0, 1, 2, 3, 4, 5, 6, 7\}$ (d) $\{9, 18, 27, 36, 45, 54, 63, 72, 81, 90\}$

CHAPTER 2

PAGE 38

1. (a) 30 (b) 39 (c) 46 (d) 19 (e) 10 (f) 25 (g) 8 (h) 34
(i) 8 (j) 10 (k) 4 (l) 39 (m) 0 (n) 17 (o) 69

2. (a) 11 (b) 16 (c) 0 (d) 0 (e) 17 (f) 99 (g) 36 (h) 26
(i) 0 (j) 0 (k) 3 (l) 20 (m) 1 (n) 100 (o) 2 (p) 936

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3. (a) 2 (b) $1\frac{7}{9}$ (c) 3 (d) 4 (e) 2 (f) 2 (g) $1\frac{5}{7}$ (h) 3
(i) 1 (j) 1 (k) 0 (l) 2 (m) 2 (n) $4\frac{1}{2}$ (o) 1

4. (a) 1 (b) 3 (c) 1 (d) 1 (e) 2 (f) 13 (g) 3 (h) 2
(i) $\frac{2}{3}$ (j) 5 (k) 3 (l) 1 (m) 16 (n) 6 (o) 2

5. (a) 1 (b) 9 (c) 15 (d) 1 (e) 4 (f) 0 (g) 7 (h) $1\frac{5}{6}$
(i) 1 (j) 6

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6. (c) yes (d) yes (e) no (f) no (g) yes (h) yes (i) no (j) yes
(k) yes (l) yes (m) yes (n) yes

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3. (a) 7 (b) 21 (c) 18 (d) 18 (e) 12 (f) 7 (g) 80 (h) 52
(i) 18 (j) 9 (k) 25 (l) 7 (m) 1 (n) 0 (o) 176 (p) 18
(q) 1 (r) 1 (s) $\frac{1}{2}$ (t) 40 (u) 162 (v) 6 (w) 54 (x) 1
(y) 8 (z) 9

PAGE 42

1. e.g. Toronto, Winnipeg, Saskatoon
(b) e.g. Detroit, New York, San Francisco
(c) As many as there are cities in Canada.
(d) Any word except names of Canadian cities. Yes.

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5. (a) {0, 1, 2, 3, 4, 5, 6}

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(b) {7, 8, 9, 10}
(c) True {7, 8, 9, 10, . . . 48, 49, 50}; False {0, 1, 2, 3, 4, 5, 6}
(d) True { }; False {0, 1, 2, 3, 4, 5, 6}

6. (a) {0, 1, 2, 3, 4, 5,} (b) {26, 27, 28, 29, 30}
(c) {10, 11} (d) {17, 18, 19, . . . 28, 29, 30}
(e) {0, 1, 2, . . . 10, 11, 12} (f) {26, 27, 28, 29, 30}
(g) {10, 11, 12, 13} (h) {23}
(i) {0, 9, 18, 27} (j) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
(k) {6} (l) { }
(m) {26} (n) {16}
(o) {10} (p) {3}
(q) {0, 5, 10, 15, 20, 25, 30} (r) {0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30}
(s) {15, 30} (t) {10, 11, 12, . . . 28, 29, 30}

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2. (a) 4, 5, 6, 7, 8, 9, 10, 11, 12 (b) 10, 12, 14, 16, 18, 20
(c) 64, 128 (d) 6, 5, 4, 3, 2,
(e) 23, 30, 38, 47 (f) 6, 7, 7, 8, 8
(g) 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7
(h) 1, 5, 1, 6, 1, 7, 1, 8, 1, 9, 1, 10, 1, 11, 1, 12, 1, 13, 1, 14, 1
(i) 19, 22, 25, 28, 31, 34, 37, 40 (j) 32, 4, 64, 4, 128, 4, 256, 4 (k) 34, 55, 89
(l) 6, 18, 7, 21, 8, 24, 9, 27, 10, 30, 11, 33, 12, 36, 13, 39, 14, 42, 15
(m) 8, 7, 6, 5, 4, 3, 2, 1 (n) 52, 65, 78, 91, 104

4. (a) 11, 12, 13 (b) 6, 4, 2 (c) 26, 29, 32 (d) 160, 320, 640
(e) 243, 729, 2187 (f) 2, 3, 1 (g) 8, 9, 8 (h) 7, 176, 7
(i) 39, 63, 102 (j) 25, 6, 30 (k) 7, 16, 7 (l) 7, 4, 6
(m) 8, 15, 8 (n) 21, 30, 20

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3. (a) {0, 6, 12, 18, . . .} (b) {10, 11, 12, 13, . . .}
(c) {5, 6, 7, 8, . . .} (d) {2, 3, 4, . . .}
(e) {0, 1} (f) {30, 31, 32, 33, . . .}
(g) {8, 9, 10, 11, . . .} (h) {0, 1, 2, . . . 22, 23, 24}
(i) {167, 168, 169 . . .} (j) {27, 28, 29, . . . 32, 33}
(k) {6} (l) {6}
(m) {25} (n) {3}
(o) {1} (p) {135}

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5. (a) {18} (b) {17} (c) {11} (d) {5} (e) {4}
(f) {87} (g) {6} (h) {0, 1, 2, 3, . . .} (i) {2}
(j) {6} (k) {7} (l) {20} (m) {0, 1, 2, 3, . . .}
(n) {0, 1, 2, 3, . . .} (o) {0, 1, 2, 3, . . .}
(p) {10} (q) {3} (r) {10} (s) { } (t) {10}

6. (a) {3} (b) {16} (c) { } (d) {0} (e) {10}
(f) {18} (g) {60} (h) {7} (i) {27} (j) {14}
(k) {23} (l) {20} (m) {11} (n) {3} (o) {5}
(p) {2} (q) {8} (r) {0, 1, 2, 3, . . .} (s) {6}
(t) {5}

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1. (a) {0, 6, 12, 18, . . .} (b) {10, 11, 12, 13, . . .} (c) {19, 20, 21, 22, . . .}
(d) {5, 6, 7, 8, . . .} (e) {9, 10, 11, 12, . . .} (f) {22}
(g) {0, 1, 2, 3, . . .} (h) {0, 1, 2, 3, . . .} (i) {0, 1, 2, 3, . . .}
(j) {0, 1} (k) {3} (l) {5}
(m) {7} (n) {14, 15, 16, 17, . . .} (o) {0, 11, 22, 33, . . .}
(p) {0, 12, 24, 36, . . .} (q) {6, 7, 8, 9, . . .} (r) { }
(s) {0, 7, 8, 9, 10, . . .} (t) {0, 1, 2, 3, . . .} (u) {0, 1, 2, 3, . . .}
(v) {0} (w) {9} (x) {0}
(y) {12, 13, 14, 15, . . .} (z) {0, 1, 2, 3, . . .}

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2. (a) {15} (b) {5} (c) {7, 8, 9, 10, . . .}
(d) {2, 3, 4} (e) {5} (f) {13}
(g) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} (h) {10, 11, 12, 13, . . .}
(i) {15, 16, 17} (j) {2} (k) {0, 1, 2, 3, . . .} (l) {3}

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3. (a) 3 (b) 84 (c) 11 (d) 10 (e) 16 (f) 26 (g) 20 (h) 28
(i) 48 (j) 90 (k) 14 (l) 20 (m) 2 (n) 1 (o) 3 (p) 3
(q) 12 (r) 1 (s) 7 (t) 1

4. (a) 10 (b) 9 (c) 5 (d) 11

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	(a)	(b)	(c)	(d)	(e)	(f)	(g)
3.	false	true	true	false	false	true	false
	true	false	false	true	true	false	true
	true	false	true	false	false	false	false
	false	true	false	true	true	true	true

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5. (a) {(7, 0), (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), (0, 7)}
(b) {(8, 0), (7, 5), (6, 10), (5, 15), (4, 20), (3, 25), (2, 30), (1, 35), (0, 40)}
(c) {(0, 13), (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1), (8, 0)}
(d) {(10, 0), (8, 1), (6, 2), (4, 3), (2, 4), (0, 5)}
(e) {(12, 0), (11, 1), (10, 2), . . . (1, 11), (0, 12)}
(f) {(7, 0), (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), (0, 7)}
(g) {(3, 0), (2, 3), (1, 6), (0, 9)}
(h) {(6, 0), (5, 1), (4, 2), (3, 3), (2, 4), (1, 5), (0, 6)}
(i) {(60, 1), (30, 2), (20, 3), (15, 4), (12, 5), (10, 6), (6, 10), (5, 12), (4, 15), (3, 20), (2, 30), (1, 60)}
(j) {(5, 0), (4, 1), (3, 2), (2, 3), (1, 4), (0, 5)}
(k) {(4, 0), (3, 1), (2, 2), (1, 3), (0, 4)}
(l) {(5, 0), (4, 1), (3, 2), (2, 3), (1, 4), (0, 5)}
(m) {(13, 0), (9, 1), (5, 2), (1, 3)}
(n) {(5, 0), (4, 1), (3, 2), (2, 3), (1, 4), (0, 5)}
(o) {(9, 0), (6, 2), (3, 4), (0, 6)}
(p) {(4, 0), (2, 3), (0, 6)}
(q) {(2, 0), (1, 1), (0, 2)}
(r) {(11, 0), (9, 1), (7, 2), (5, 3), (3, 4), (1, 5)}

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7. (a) {(3, 0), (4, 1), (5, 2), . . .}
(b) {(16, 0), (15, 1), (1,4 2), . . . (2, 14), (1, 15), (0, 16)}
(c) {(10, 0), (9, 1), (8, 2), . . . (1, 9), (0, 10)}
(d) {(0, 6), (1, 8), (2, 10), . . . (1, 8), (0, 6)}
(e) {(0, 0), (1, 1), (2, 2), . . .}
(f) {(32, 1), (16, 2), (8, 4), (4, 8), (2, 16), (1, 32)}
(g) {(0, 0), (2, 1), (4, 2), . . .}
(h) {(0, 2), (1, 1), (2, 0)}
(i) {(0, 3), (1, 4), (2, 5), . . .}
(j) {(2, 0), (3, 1), (4, 2), . . .}
(k) {(0, 9), (1, 8), (2, 7), . . . (1, 8), (0, 9)}
(l) {(3, 0), (4, 1), (5, 2), . . .}
(m) {(2, 0), (3, 1), (4, 2), . . .}
(n) {(10, 0), (9, 1), (8, 2), . . . (1, 9), (0, 10)}
(o) {(2, 0), (3, 1), (4, 2), . . .}
(p) {(0, 0), (1, 1), (2, 2), . . .}
(q) {(48, 1), (24, 2), (16, 3), (12, 4), (8, 6), (6, 8), (4, 12), (3, 16), (2, 24), (1, 48)}
(r) {(0, 0), (1, 1), (2, 2), . . .}
(s) {(1, 1), (2, 3), (3, 5), . . .}

(t) {(4, 0), (3, 0), (2, 0), (1, 0), (0, 0), (3, 1), (2, 1), (1, 1), (0, 1), (2, 2), (1, 2), (0, 2), (1, 3), (0, 3), (0, 4)}
(u) {(6, 0), (7, 1), (8, 2), . . .} (v) {(0, 0)}
(w) {(0, 7), (1, 8), (2, 9), . . .} (x) {(0, 0), (1, 1), (2, 2), . . .}
(y) {(0, 5), (1, 6), (2, 7), . . .} (z) all but (0, 0)

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9. (a) ▨ + ▲ = 75 (b) ▨ × ▲ = 300 (c) ▨ = ▲ + 6
(d) 2 × ▨ + 2 × ▲ = 300 (e) ▨ = 3 × ▲ (f) ▨ = 9 × ▲
(g) 5▨ = 4▲ (h) ▨ = 10 × ▲ (i) ▨ = ▲ − 5
(j) T = ▨ + (2 × ▲)

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2. (a) {4} (b) {(3, 0), (4, 1), (5, 2), . . .}
(c) {(6, 1), (3, 2), (2, 3), (1, 6)} (d) { }
(e) {(11, 0), (10, 1), (9, 2), . . . (1, 10), (0, 11)}
(f) {(1, 0), (2, 2), (3, 4), . . .} (g) {0, 1, 2, 3, 4, 5, 6}
(h) {10, 11, 12, 13, . . .} (i) {0, 1, 2, 3}
(j) {8, 9, 10, 11, . . .} (k) {(0, 0), (1, 2), (2, 4), . . .}
(l) {(0, 0), (1, 1), (2, 2), . . .} (m) {0}
(n) {(12, 0), (11, 1), (10, 2), . . . (1, 11), (0, 12)}
(o) {(0, 0), (2, 1), (4, 2), (6, 3), . . .}
(p) {(0, 1), (0, 2), (0, 3), . . . (1, 3), (1, 4), (1, 5), . . . (2, 5), (2, 6), (2, 7), . . . (3, 7), (3, 8), (3, 9), . . .} and so on
(q) {(0, 0), (3, 2), (6, 4), (9, 6), . . .}
(r) {(0, 7), (0, 8), (0, 9), . . . (1, 6), (1, 7), (1, 8), . . . (2, 5), (2, 6), (2, 7), . . . (3, 4), (3, 5), (3, 6), . . . (4, 3), (4, 4), (4, 5), . . . (5, 2), (5, 3), (5, 4), . . . (6, 1), (6, 2), (6, 3), . . . (7, 0), (8, 0), (9, 0), . . .}
(s) {0} (t) {0, 1} (u) {0, 1, 2, 3, 4} (v) {6}
(w) {0} (x) {(6, 0), (6, 1), (6, 2), . . .}

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4. (a) ▨ × 3 + 5 = 29; ▨ = 8 (b) ▨ × 4 + 6 = 50; ▨ = 11
(c) (▨ + 15) × 2 = 360; ▨ = 165 (d) 4 × ▨ + 3 × ▲ = 133
(i) {(1, 43), (4, 39), (7, 35), (10, 31), (13, 27), (16, 23), (19, 19), (22, 15), (25, 11), (28, 7), (31, 3)} (ii) {(19, 19)}

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3. (a) 26 gal. (b) 12,314 girls (c) 20 pints (d) 73 acres
(e) 34 fish (f) 2 mi. 1080 yd. (g) 150 days (h) 27 goals
(i) 9¢ (j) 39 (k) 36 eggs (l) $2.40 (m) 220 yd. (n) $12.57 (o) 7,500

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2. (a) 100 lb. (b) 8 ft. (c) 6 oz. (d) 9 boys

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2. (a) 13 (b) 8 (c) 16 (d) 11 (e) 3 (f) 66 (g) 1
(h) 9 (i) 8 (j) 0 (k) 3 (l) $3\frac{3}{5}$ (m) 0 (n) 114
(o) 3 (p) 1 (q) 1 (r) 2 (s) 1 (t) 0

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3. (a) {4} (b) { } (c) {17} (d) {11}
(e) {0, 1, 2, 3, 4, 5, 6} (f) {10, 11, 12, 13, . . .}
(g) {(0, 5), (0, 4), (0, 3), (0, 2), (0, 1), (0, 0), (1, 4), (1, 3), (1, 2), (1, 1), (1, 0), (2, 3), (2, 2), (2, 1), (2, 0), (3, 2), (3, 1), (3, 0), (4, 1), (4, 0), (5, 0)}

(h) {3} (i) {7} (j) {9, 10, 11} (k) {0, 1, 2, 3, . . .} (l) { }
(m) {0, 1, 2, 3, . . .} (n) {8, 9, 10, 11, . . .} (o) {11} (p) {0, 1}
(q) {11} (r) {9} (s) {0, 1, 2, 3, 4, . . .} (t) {0, 1, 2, 3, . . .}

4. (a) {(11, 0), (10, 1), (9, 2), . . . (2, 9), (1, 10), (0, 11)}
(b) {(16, 0), (14, 1), (12, 2), . . . (2, 7), (0, 8)}
(c) {(3, 0, (4, 1), (5, 2), . . .}
(d) {(2, 0), (3, 2), (4, 4), . . .}
(e) {(16, 1), (8, 2), (4, 4), (2, 8), (1, 16)}
(f) {(0, 0), (1, 1), (2, 2), . . .}
(g) {(18, 0), (17, 1), (16, 2), . . . (1, 17), (0, 18)}
(h) {(0, 0), (1, 2), (2, 4), . . .}
(i) {(0, 14), (1, 13), (2, 10), (3, 5)}
(j) {(37, 0), (36, 1), (33, 2), (28, 9), (21, 4), (12, 5), (1, 6)}
(k) {(0, 0), (1, 1), (2, 2), . . .}
(l) {(0, 0)}
(m) {(7, 8), (8, 23), (9, 40), (10, 59), . . .}
(n) {(10, 0), (8, 6), (6, 8), (0, 10)}
(o) {(4, 1), (1, 3)}
(p) {(3, 1), (1, 4)}
(q) {(0, 0), (3, 2), (6, 4), (9, 6), . . .}
(r) {(1, 3), (4, 2), (7, 1), (10, 0)}
(s) {(0, 19), (1, 15), (2, 11), (3, 7), (4, 3)}
(t) {1, 3), (2, 2)}

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5. (a) 9 (b) 13 (c) 4 (d) 5 (e) 4 (f) 441 (g) 1
(h) $\frac{2}{3}$ (i) 1 (j) 9

1. (a) {(0, 1, 2, 3, . . . 14, 15, 16} (b) {0, 1, 2, 3, 4, 5, 6, 7, 8}
(c) {0, 1, 2, 3, 4, 5, 6, 7, 8} (d) {46, 47, 48, 49, . . .}
(e) {2, 3, 4, 5, . . .} (f) {0, 1, 2, 3, 4, 5, 6, 7}
(g) {2, 3, 4, 5, . . .} (h) {1, 2, 3, 4}
(i) {1} (j) {0, 1, 2, 3, . . .}
(k) {0, 1, 2, 3, 4, 5} (l) {2, 3, 4, 5, . . .}
(m) {0, 1, 2, 3, 4, 5,} (n) {5, 6, 7, 8, . . .}
(o) {11, 12, 13, 14, . . .} (p) {0, 1, 2, 3, 4, 5}

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3. (b) 10 (c) 45 (d) 26 (e) 31 (f) 13 (g) 18 (h) 0

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(i) 1 (j) 3 (k) 33 (l) 24 (m) 3 (n) 3 (o) 6
(p) $3\frac{5}{7}$ (q) 1 (r) 5 (s) 4 (t) 2 (u) 2

4. (a) {(6, 1), (3, 2), (2, 3), (1, 6)} (b) {(3, 1), (4, 3), (5, 5), . . .}
(c) {(7, 0), (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), (0, 7)}
(d) {(3, 0), (5, 1), (7, 2), (9, 3), . . .} (e) {(7, 0), (8, 1), (9, 2), . . .}
(f) {(3, 0), (4, 1), (5, 2), (6, 3), . . .} (g) {(0, 0), (1, 0), (2, 0), (3, 0), . . .}
(h) {(6, 1), (3, 2), (2, 3), (1, 6)} (i) {(0, 0), (1, 1), (2, 2), (3, 3), . . .}
(j) {(0, 0), (1, 1), (2, 2), (3, 3), . . .} (k) {(0, 0), (1, 0), (2, 0), (3, 0), . . .}
(l) { } (m) {(1, 0), (2, 1), (3, 2), (4, 3), . . .}
(n) { } (o) {(3, 0), (4, 1), (5, 2), (6, 3), . . .}
(p) {(0, 0)} (q) {(6, 1), (3, 2), (2, 3), (1, 6)}
(r) {(0, 23), (1, 20), (2, 17), (3, 14), (4, 11), (5, 8), (6, 5), (7, 2)}
(s) {(8, 0), (9, 1), (10, 2), (11, 3), . . .} (t) {(0, 1), (6, 2), (12, 3), (18, 4), . . .}
(u) {(0, 2), (1, 3), (2, 4), . . .} (v) {(0, 12), (3, 7), (6, 2)}

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2. a; b; d; f; g.

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4. b; d; f.

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6. (a) 18 (b) 16 (c) 15 (d) 20 (e) 20 (f) 14 (g) 15 (h) 23
(i) 24 (j) 30 (k) 17 (l) 30 (m) 29 (n) 29 (o) 19 (p) 27
(q) 30 (r) 17 (s) 20 (t) 27 (u) 20 (v) 25 (w) 25 (x) 29

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9. (a) 70 (b) 300 (c) 700 (d) 90 (e) 600 (f) 210 (g) 800 (h) 700
(i) 700 (j) 1100 (k) 240 (l) 21,000 (m) 90 (n) 600 (o) 210
(p) 800 (q) 360 (r) 1200 (s) 1050 (t) 3300 (u) 160 (v) 360 (w) 360
(x) 5400

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13. (a) 100 (b) 120 (c) 93 (d) 130 (e) 60 (f) 133 (g) 107 (h) 101
(i) 160 (j) 150 (k) 210 (l) 199

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15. (a) 120 (b) 140 (c) 160 (d) 90 (e) 72 (f) 130 (g) 70 (h) 110
(i) 64 (j) 330 (k) 210 (l) 410

17. (a) 48 (b) 29 (c) 153 (d) 147 (e) 64 (f) 56 (g) 121 (h) 118
(i) 81 (j) 169 (k) 158 (l) 115

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18. (a) {7} (b) {21} (c) {19} (d) {13} (e) {9} (f) {4} (g) {4} (h) {15}
(i) {11} (j) {17} (k) {0, 1, 2, 3, . . .} (l) {7} (m) {8}
(n) {0, 1, 2, 3, . . .} (o) }10 (p) }16{ (q) {7} (r) {36}
(s) {0, 1, 2, 3, . . .} (t) {16}

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22. (a) 72 (b) 65 (c) 126 (d) 90 (e) 128

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24. (a) 180 (b) 448 (c) 275 (d) 377 (e) 684

26. (a) Add'n is commutative {3} (b) Add'n is associative {7}
(c) Distributive {2} (d) Add'n is associative {7}
(e) Distributive {10}

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2. (a) 199,920 (b) 4,278,880 (c) 6,385,570 (d) 1,748,160
(e) 193,125 (f) 4,196,258 (g) 7,122,112 (h) 2,194,821
(i) 356,658 (j) 22,187,286 (k) 4,564,755 (l) 68,913,166

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6. (a) 237 r 14 (b) 105 r 28 (c) 256 r 33 (d) 171 r 1
(e) 162 r 3 (f) 462 r 78 (g) 144 r 232 (h) 43 r 594
(i) 205 r 168 (j) 287 r 30 (k) 325 r 134 (l) 776 r 59

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9. (a) 82 (b) 75 (c) 309 (d) 175 r 2
(e) 274 r 2 (f) 19 (g) 19 (h) 91
(i) 90 r 3 (j) 1141 r 4 (k) 468 r 4 (l) 888 r 2
(m) 689 r 1 (n) 722 r 1 (o) 804 r 2 (p) 6645 r 2
(q) 9002 (r) 8589 (s) 902 r 3 (t) 554 r 1
(u) 15750 r 1 (v) 1134 r 5 (w) 1363 r 2 (x) 2000 r 1

CHAPTER TEST

1. (a) {British Columbia, Manitoba, Ontario, Nova Scotia, Newfoundland}
(b) {17, 18, 19, 20, . . .} (c) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

2. (a) {10, 12, 14, 16, 18} (b) {8, 7, 6, 5, 4, 3}
(c) {1, 13, 1, 14, 1, 15, 1, 16, 1, 17, 1, 18, 1} (d) {136, 220}

3. (a) 20 (b) 168 (c) 49 (d) 5 (e) 4 (f) 45 (g) 7 (h) 4
(i) 12 (j) 10 (k) 22 (l) 1 (m) 33 (n) $\frac{1}{2}$ (o) 7

4. (a) {0, 1, 2, 3, 4} (b) {0, 1, 2, 3, 4, 5} (c) {12, 13, 14} (d) {19}
(e) {9} (f) {19} (g) {7, 8, 9, 10, . . .} (h) {4} (i) {6} (j) {8}

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5. (a) ▨×▨ is between 400 and 500 {21, 22}
(b) ▨×8−7=97 {13} (c) ▨×5−▨=280 {70}

6. (a) {(0, 12), (1, 10), (2, 8), (3, 6), (4, 4), (5, 2), (6, 0)}
(b) {(3, 8), (5, 5), (7, 2)} (c) {(9, 0), (10, 1), (11, 2), (12, 3)}
(d) {(5, 0), (10, 1), (15, 2), (20, 3), . . .}

7. (a) {4} (b) {11} (c) {7} (d) {9} (e) {115} (f) {0, 1, 2, 3, . . .}

8. (a) {(9, 0), (8, 1), (7, 2), (6, 3), (5, 4), (4, 5), (3, 6), (2, 7), (1, 8), (0, 9)}
(b) {(2, 1), (3, 3), (4, 5), . . .} (c) {(3, 1)}
(d) {(6, 0), (7, 1), (8, 2), . . .} (e) {(1, 4), (5, 3), (9, 2), (13, 1), (17, 0)}
(f) {(0, 1), (1, 0)}

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9. (a) {7} distributive (b) {7} add'n is associative
(c) {6} mult'n is associative (d) {11} mult'n is associative

10. (a) {12} (b) {16} (c) {5} (d) {5} (e) {3} (f) {10} (g) {10} (h) {0, 1}

CHAPTER 3

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2. (a) 12; 42; 102 (b) 7; 91; 287; 1001; 6006; 10,010
(c) 200; 5000; 6025; 4075 (d) 2; 16; 1006; 246; 162
(e) 1; 97; 116; 414; 813; 1116; 1021; (f) 36; 45; 2133; 7119
(g) 1; 13 (h) 1; 2; 3; 4; 5; 6; 10; 12
(i) 15; 20; 30; 60 (j) 1; 2; 4; 5; 10; 20; 40; 80
(k) 1; 7; 11; 13; 23 (l) 1

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4. (a) 1, 7 (b) 1, 2, 4, 8 (c) 1, 2, 5, 10 (d) 1, 2, 4, 8, 16
(e) 1, 2, 3, 5, 6, 10, 15, 30 (f) 1, 2, 4, 5, 8, 10, 20, 40 (g) 1, 71
(h) 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90 (i)1, 2, 4, 5, 10, 20, 25, 50, 100
(j) 1,101 (k) 1, 7, 13, 91 (l) 1, 2, 4, 8, 11, 22, 44, 88

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2. (a)17, 19, 23, 29, 31, 37 (b) 41; 43; 47; 59; 67 (c) 5 (d) 3
(e) 2 (f) no. See definition of primes.

4. (a) 16; 18; 20; 21; 22; 24 (b) 33; 63; 93 (c) no.

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6. (a) 2×2×3; 2×2×2×2; 2×3×3; 5×5; 3×3×3; 1×29; 2×2×3×3; 2×3×7
(b) 2×2; 2×2×2×2×2; 2×2×2×2; 2×2×2×2×2×2; 2×1; 2×2×2; 2×2×2×2×2×2×2 All have 2 as a factor.
(c) 1×17; 1×31; 1×41; 1×59; 1×61; 1×101 All are prime.
(d) 4×1, 2×2; 25×1, 5×5; 1×1; 2×2×3×3; 2×2×2×2×2×2; 3×3; 7×7; 2×2×2×2 All are perfect squares.
(e) 2×2×2; 3×3×3; 4×4×4; 5×5×5; 6×6×6; 7×7×7; 1×1×1
All are cubes.

7. (b) 2 (c) 11 (d) 5 (e) 5 (f) 19 (g) 2 (h) 7 (i) 2
(j) 7

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2. (a) (i) no (ii) yes (iii) no (iv) no
(b) (i) yes (ii) yes (iii) no (iv) no

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3. (a) (i) 6 (ii) 21 (iii) 14 (b) (i) 35 (ii) 21 (iii) 15
(c) (i) 143 (ii) 39 (iii) 33 (d) (i) 51 (ii) 9
(e) (i) 22 (ii) 4 (f) (i) 39 (ii) 9
(g) (i) 84 (ii) 196 (iii) 294

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5. (a) 13 (b) 21 (c) 26 (d) 49 (e) 17 (f) 253 (g) 18 (h) 70
(i) 55 (j) 231

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7. (b) {1, 3, 7, 9, 21, 63} (c) {1, 2, 4, 7, 14, 28}
(d) {1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200}
(e) {1, 2, 4, 8, 11, 22, 44, 88} (f) {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
(g) {1, 2, 3, 4, 6, 9, 12, 18, 36} (h) {1, 3, 7, 9, 21, 49, 63, 147, 441}
(i) {1, 173} (j) {1, 2, 3, 6, 19, 38, 57, 114}
(k) {1, 2, 3, 6, 9, 17, 18, 34, 51, 102, 153, 306}
(l) {1, 3, 5, 9, 15, 27, 45, 135} (m) {1, 2, 7, 14, 49, 98, 343, 686}
(n) {1, 2, 4, 5, 8, 10, 16, 20, 40, 80} (o) {1, 2, 3, 6, 9, 18, 27, 54, 81, 162}
(p) {1, 3, 5, 7, 11, 15, 21, 33, 35, 55, 77, 105, 165, 231, 385, 1155}
(q) {1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150}
(r) {1, 2, 5, 10, 25, 50, 125, 250} (s) {1, 3, 5, 15, 23, 69, 115, 345}

(t) {1, 3, 5, 7, 9, 15, 21, 35, 45, 63, 105, 315}
(u) {1, 2, 4, 8, 16, 32, 64} (v) {1, 5, 13, 65}
(w) {1, 937} (x) {1, 7, 11, 13, 17, 77, 91, 119, 143, 187, 221, 1001, 1309, 1547, 2431, 17017} (y) {1, 5, 25, 43, 215, 1075}
(z) {1, 2, 3, 4, 6, 8, 9, 11, 12, 16, 18, 22, 24, 33, 36, 44, 48, 66, 72, 88, 99, 132, 144, 176, 198, 264, 396, 528, 792, 1584}

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2. (a) 11 (b) 8 (c) 2 (d) 11 (e) 27 (f) 23 (g) 69 (h) 9
(i) 8 (j) 7 (k) 130 (l) 133 (m) 202 (n) 12 (o) 80 (p) 9
(q) 160 (r) 35

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2. (a) {1, 3} (b) {1, 5} (c) {1, 7} (d) {1, 11}
(e) {1, 5} (f) {1, 3} (g) {1, 3} (h) {1, 13}
(i) {1, 17} (j) {1, 23} (k) {1} (l) {1, 2, 4}
(m) {1, 2} (n) {1, 7} (o) {1, 13} (p) {1, 19}
(q) {1, 23} (r) {1, 7} (s) {1, 2, 4} (t) {1, 2, 5, 10}
(u) {1, 2, 4, 8}

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4. (b) {1, 2, 3, 6} (c) {1, 2, 4} (d) {1, 3, 9} (e) {1, 5, 25}
(f) {1, 2, 5, 10} (g) {1, 2, 3, 4, 6, 12} (h) {1, 2, 4, 8, (i) {1, 2}

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2. (a) 1 (b) 5 (c) 16 (d) 15 (e) 50 (f) 3 (g) 13 (h) 1
(i) 1 (j) 2 (k) 4 (l) 5

2. b; d; e; g; l; m; n; o

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3. (a) 35, 70, 105, 140 (b) 18, 36, 54, 72 (c) 24, 48, 72, 96 (d) 6, 12, 18, 24
(e) 70, 140, 210, 280 (f) 24, 48, 72, 96 (g) 20, 40, 60, 80
(h) 24, 48, 72, 96 (i) 60, 120, 180, 240 (j) 10, 20, 30, 40
(k) 36, 72, 108, 144 (l) 77, 154, 231, 308 (m) 60, 120, 180, 240
(n) 34, 68, 102, 136 (o) 144, 288, 432, 576

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2. (a) 24 (b) 30 (c) 14 (d) 30 (e) 36 (f) 40 (g) 36 (h) 18
(i) 72 (j) 12 (k) 15 (l) 30 (m) 60 (n) 132 (o) 3

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5. (b) 60 (c) 48 (d) 126 (e) 198 (f) 120 (g) 120 (h) 280 (i) 420
(j) 98 (k) 165 (l) 240 (m) 288 (n) 360 (o) 540 (p) 200 (q) 260
(r) 72 (s) 210 (t) 2310 (u) 1078 (v) 64

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8. (a) 60 (b) 180 (c) 105 (d) 210 (e) 180 (f) 300 (g) 300 (h) 180
(i) 200 (j) 216 (k) 600 (l) 990 (m) 120 (n) 60 (o) 2520

9. (a) 9, 18, 27, 36, 45, 54, 63, etc. (b) {1, 2, 4, 5, 10, 20, 25, 50, 100}
(c) 83, 89, 97 (d) 2×2×2×2×3×5
(e) 2×2×2×3×5×5 (f) $\frac{3}{11}$ (g) $\frac{17}{63}$ (h) 7, 7, 7

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(i) {3, 3} (j) {1, 2, 3, 5, 6, 10, 15, 30}
(k) {1, 2, 4, 5, 8, 10, 20, 40} (l) {1, 2, 4, 7, 8, 14, 28, 56}
(m) {1, 47} (n) {1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84}
(o) {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300}
(p) {1, 2, 5, 7, 10, 14, 35, 70} (q) 19 (r) 7 (s) 3 r 14
(t) 180 (u) 36 (v) 54 (w) 72 (x) 108

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13. (b) $(3\times10^2)+(9\times10)+(2\times1)$ (c) $(5\times10^2)+(8\times10)+(0\times1)$
(d) $(7\times10^2)+(8\times10)+(6\times1)$
(e) $(1\times10^3)+(3\times10^2)+(5\times10)+(8\times1)$
(f) $(1\times10^4)+(0\times10^3)+(0\times10^2)+(8\times10)+(4\times1)$
(g) $(7\times10^5)+(2\times10^4)+(0\times10^3)+(3\times10^2)+(1\times10)+(6\times1)$
(h) $(2\times10^5)+(6\times10^4)+(5\times10^3)+(0\times10^2)+(0\times10)+(0\times1)$
(i) $(1\times10^6)+(0\times10^5)+(0\times10^4)+(0\times10^3)+(0\times10^2)+(0\times10)+(0\times1)$

2. (a) 4^3 (b) 5^5 (c) 7^4 (d) 9^2 (e) 26^3 (f) 19^3 (g) 10^4 (h) 1^5
(i) 6^4

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3. (b) 5×5 (c) $11\times11\times11$ (d) $9\times9\times9\times9$ (e) 8×8
(f) $7\times7\times7\times7\times7$ (g) $3\times3\times3\times3$ (h) $18\times18\times18$ (i) 26×26
(j) $19\times19\times19$ (k) $20\times20\times20\times20$ (l) $4\times4\times4\times4$

4. (b) 256 (c) 27 (d) 49 (e) 64 (f) 243 (g) 10,000 (h) 121
(i) 1728 (j) 729 (k) 169 (l) 1296

5. (b) $7\times7=7^2$ (c) $2\times2\times2\times2\times2=2^5$ (d) $2\times2\times2\times2\times2\times2=2^6$
(e) $11\times11=11^2$ (f) $6\times6=6^2$ (g) $3\times3\times3=3^3$ (h) $5\times5\times5=5^3$
(i) $3\times3\times3\times3=3^4$

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4. (a) 2 (b) 5 (c) 9 (d) 7 (e) 6 (f) 1 (g) 10 (h) 8
(i) 12 (j) 11 (k) 0 (l) 30 (m) 25 (n) 13 (o) 20 (p) 15

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6. (a) 15 (b) 16 (c) 25 (d) 9 (e) 8 (f) 33 (g) 24 (h) 17
(i) 35 (j) 40 (k) 50 (l) 32 (m) 42 (n) 48 (o) 21 (p) 36
(q) 39 (r) 70

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9. (a) 83 (b) 67 (e) 97 (f) 71 (j) 127

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11. (a) 331 (e) 61 (g) 191 (l) 251 (r) 607
12. 211; 223; 227; 229; 233; 239

CHAPTER TEST

1. {7, 11, 13} 2. 9 3. 401; 409; 419; 421; 431; 433

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4. (a) $2\times2\times2\times3\times3$ (b) $5\times3\times3$
(c) $5\times5\times5\times2$ (d) $2\times2\times2\times2\times3\times3\times5$
5. (a) 13 (b) 11 (c) 7 (d) 31
6. (a) yes (b) no (c) yes (d) yes

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7. (a) 19 (b) 63 (c) 39 (d) 125
8. (a) 12 (b) 15 (c) 34 (d) 25 (e) 14 (f) 18
9. (a) 70 (b) 80 (c) 42 (d) 240 (e) 210 (f) 840
10. (a) 6 (b) 20 (c) 42 (d) 28 (e) 56 (f) 43 (g) 36 (h) 60
11. Factors of 1008: {1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 24, 28, 36, 42, 48, 56, 63, 72, 84, 112, 126, 144, 168, 252, 336, 504, 1008}
    Factors of 792: {1, 2, 3, 4, 6, 8, 9, 11, 12, 18, 22, 24, 33, 36, 44, 66, 72, 88, 99, 132, 198, 264, 396, 792}
    Common factors: {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}
    Greatest common factor: 72
12. a; b; g; h; i; j

CHAPTER 4

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1. (a) tens (b) ones (c) millions (d) hundred-thousands
   (e) ones (f) ten-thousands (g) thousands
   (h) hundreds (i) hundreds (j) ten-thousands
2. (b) (2×1000)+(3×100)+(1×10)+(5×1)
   (c) (9×10,000)+(1×1000)+(2×100)+(1×10)+(6×1)
   (d) (4×100,000)+(2×10,000)+(0×1000)+(0×100)+(0×10)+(0×1)
   (e) (1×1,000,000)+(0×100,000)+(0×10,000)+(0×1000)+(1×100)+(0×10)+(0×1)
   (f) (4×1,000,000)+(2×100,000)+(9×10,000)+(6×1000)+(3×100)+(2×10)+(7×1)
   (g) (6×10)+(4×1)
   (h) (2×100,000)+(0×10,000)+(0×1000)+(1×100)+(5×10)+(3×1)
   (i) (5×10,000,000)+(3×1,000,000)+(1×100,000)+(6×10,000)+(4×1000)+(2×100)+(3×10)+(2×1)
3. (b) $(4\times10^2)+(1\times10)+(6\times1)$ (c) $(1\times10^3)+(0\times10^2)+(9\times10)+(5\times1)$
   (d) $(1\times10^4)+(7\times10^3)+(3\times10^2)+(2\times10)+(5\times1)$
   (e) $(4\times10^5)+(1\times10^4)+(6\times10^3)+(8\times10^2)+(3\times10)+(3\times1)$
   (f) $(6\times10^6)+(0\times10^5)+(1\times10^4)+(4\times10^3)+(0\times10^2)+(3\times10)+(7\times1)$
   (g) $(2\times10^6)+(0\times10^5)+(9\times10^4)+(8\times10^3)+(6\times10^2)+(3\times10)+(7\times1)$
   (h) $(1\times10^7)+(0\times10^6)+(0\times10^5)+(0\times10^4)+(0\times10^3)+(0\times10^2)+(0\times10)+(0\times1)$
   (i) $(2\times10^6)+(0\times10^5)+(1\times10^4)+(4\times10^3)+(9\times10^2)+(3\times10)+(7\times1)$

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6. 22_5, 40_5, 2_5, 34_5, 3_5, 43_5, 44_5, 31_5, 24_5, 30_5
7. 7, 13, 4, 15, 19, 14, 9, 22, 24, 5
8. (1×5)+(2×1) (2×5)+(3×1) (4×1) (3×5)+(0×1)
   (3×5)+(4×1) (2×5)+(4×1) (1×5)+(4×1) (4×5)+(2×1)
   (4×5)+(4×1) (1×5)+(0×1)

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11. $(4\times5^2)+(1\times5)+(3\times1)$ $(2\times5^2)+(0\times5)+(3\times1)$ $(1\times5^2)+(0\times5)+(0\times1)$
    $(4\times5^2)+(4\times5)+(0\times1)$ $(3\times5^2)+(4\times5)+(3\times1)$ $(4\times5^2)+(0\times5)+(2\times1)$

12. 108; 53; 25; 120; 98; 102.

14. 25; 125; 625; 3125; 15,625; 78,125

15. (a) $(1\times5^3)+(0\times5^2)+(1\times5)+(0\times1)$ (b) $(3\times5^3)+(4\times5^2)+(3\times5)+(2\times1)$
(c) $(4\times5^3)+(0\times5^2)+(0\times5)+(0\times1)$ (d) $(1\times5^3)+(2\times5^2)+(3\times5)+(0\times1)$
(e) $(4\times5^4)+(0\times5^3)+(1\times5^2)+(3\times5)+(4\times1)$
(f) $(1\times5^4)+(0\times5^3)+(0\times5^2)+(0\times5)+(0\times1)$
(g) $(3\times5^4)+(2\times5^3)+(1\times5^2)+(0\times5)+(3\times1)$
(h) $(2\times5^4)+(0\times5^3)+(2\times5^2)+(0\times5)+(2\times1)$

16. (a) 130 (b) 492 (c) 500 (d) 190 (e) 2544 (f) 625 (g) 2153 (h) 1302

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18. (a) 141_5 (b) 304_5 (c) 1324_5 (d) 1000_5
(e) 120_5 (f) 10241_5 (g) 10340_5 (h) 13142_5
(i) 14331_5 (j) 43332_5 (k) 32044_5 (l) 44000_5

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1. (a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 1 (g) 2
(h) 3 (i) 4 (j) 2 (k) 3 (l) 4 (m) 10_5 (n) 3
(o) 4 (p) 10_5 (q) 4 (r) 10_5 (s) 11_5 (t) 11_5 (u) 10_5
(v) 11_5 (w) 12_5 (x) 12_5 (y) 13_5

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4. (a) 34_5 (b) 43_5 (c) 44_5 (d) 100_5 (e) 33_5

6. 41_5; 103_5; 110_5; 102_5; 110_5; 131_5

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8. 121_5; 112_5; 121_5; 113_5; 121_5; 100_5

10. 423_5; 342_5; 1320_5; 1040_5; 1000_5; 434_5; 1004_5; 341_5; 1210_5; 1310_5

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11. (b) 10210_5 (c) 11214_5 (d) 13203_5 (e) 12001_5
(f) 13101_5 (g) 12124_5 (h) 3003_5 (i) 11343_5
(j) 4120_5 (k) 4313_5 (l) 11040_5 (m) 4443_5

13. (a) 2_5 (b) 1_5 (c) 3_5 (d) 4_5

14. (b) 3_5 (c) 10_5 (d) 4_5 (e) 4_5 (f) 2_5
(g) 4_5 (h) 3_5 (i) 10_5 (j) 11_5 (k) 12_5

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16. (a) 4_5 (b) 241_5 (c) 32_5 (d) 104_5 (e) 24_5 (f) 144_5
(g) 23_5 (h) 12_5 (i) 34_5 (j) 0 (k) 214_5 (l) 44_5

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18. (a) 22_5 (b) 141_5 (c) 2014_5 (d) 411_5 (e) 2114_5 (f) 1344_5
(g) 2324_5 (h) 231_5 (i) 342_5 (j) 2334_5 (k) 1000_5 (l) 434_5

3. 25_7 4. 10, 5, 7 5. Base 7; 6-ones; 2-sevens; 4-forty-nines

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9. (a) 40_7 (b) 104_7 (c) 100_7 (d) 103_7 (e) 52_7 (f) 312_7
(g) 261_7 (h) 460_7 (i) 351_7 (j) 1014_7 (k) 620_7 (l) 504_7
(m) 325_7 (n) 1462_7 (o) 1160_7 (p) 1014_7 (q) 1036_7 (r) 1015_7

(s) 1224_7 (t) 1112_7 (u) 1553_7 (v) 3062_7 (w) 6062_7 (x) 12643_7
(y) 15103_7

10. (a) 21_7 (b) 21_7 (c) 51_7 (d) 10_7 (e) 11_7 (f) 26_7
(g) 16_7 (h) 11_7 (i) 34_7 (j) 26_7 (k) 325_7 (l) 464_7
(m) 116_7 (n) 315_7 (o) 255_7 (p) 264_7 (q) 65_7 (r) 234_7
(s) 54_7 (t) 303_7 (u) 356_7 (v) 163_7 (w) 2366_7 (x) 342_7
(y) 2325_7 (z) 656_7

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2. (a) 35_7 (b) 63_7 (c) 120_7 (d) 100_7 (e) 222_7
(f) 1000_7 (g) 1001_7 (h) 2102_7 (i) 2626_7 (j) 1100_7
(k) 10000_7 (l) 20000_7 (m) 40056_7 (n) 2062_7 (o) 4543_7
(p) 5605_7 (q) 60215_7 (r) 54032_7 (s) 13000_7 (t) 4661_7
(u) 4264_7 (v) 6666_7 (w) 41302_7 (x) 3000_7 (y) 44446_7

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4. (a) 13 (b) 17 (c) 33 (d) 41 (e) 7
(f) 42 (g) 49 (h) 61 (i) 223 (j) 104
(k) 306 (l) 275 (m) 343 (n) 687 (o) 588
(p) 508 (q) 1462

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6. (a) (i) 16 (ii) 22_7 (iii) 31_5 (b) (i) 23 (ii) 32_7 (iii) 43_5
(c) (i) 21 (ii) 30_7 (iii) 41_5 (d) (i) 52 (ii) 103_7 (iii) 202_5
(e) (i) 10 (ii) 13_7 (iii) 20_5 (f) (i) 80 (ii) 143_7 (iii) 310_5
(g) (i) 90 (ii) 156_7 (iii) 330_5

CHAPTER TEST

1. 1_5
2_5
3_5
4_5
10_5
11_5
12_5
13_5
14_5
20_5
30_5
100_5
200_5
300_5
400_5
420_5
1000_5
1030_5
1100_5
10000_5

2. (a) 13120_5 (b) 3000_5
(c) 20320_5 (d) 10321_5
(e) 1240_5 (f) 1403_5
(g) 1220_5 (h) 1134_5

3. (a) 111_5 (b) 10_5
(c) 444_5 (d) 33_5
(e) 2444_5 (f) 134_5
(g) 2004_5 (h) 214_5

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4. (b) 11_5 (c) 14_5 (d) 4_5 (e) 13_5 (f) 4_5 (g) 31_5

5. 0_5 0_5 0_5 0_5 0_5
0_5 1_5 2_5 3_5 4_5
0_5 2_5 4_5 11_5 13_5
0_5 3_5 11_5 14_5 22_5
0_5 4_5 13_5 22_5 31_5

6. (a) 623_7 (b) 623_7 (c) 1105_7 (d) 1360_7 (e) 605_7
(f) 564_7 (g) 632_7 (h) 1000_7 (i) 1163_7 (j) 2101_7

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7. (a) 101_7 (b) 150_7 (c) 361_7 (d) 116_7 (e) 156_7
(f) 463_7 (g) 2510_7 (h) 1526_7 (i) 1513_7 (j) 155_7

8. (a) (i) 13 (ii) 16_7 (iii) 23_5 (b) (i) 21 (ii) 30_7 (iii) 41_5
(c) (i) 42 (ii) 60_7 (iii) 132_5 (d) (i) 70 (ii) 130_7 (iii) 240_5

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9. (a) $(6\times 7^3)+(0\times 7^2)+(1\times 7)+(4\times 1)$
(d) $(3\times 7^4)+(2\times 7^3)+(0\times 7^2)+(5\times 7)+(5\times 1)$
(c) $(6\times 7^4)+(1\times 7^3)+(4\times 7^2)+(3\times 7)+(2\times 1)$
(d) $(6\times 7^3)+(0\times 7^2)+(6\times 7)+(0\times 1)$
(e) $(3\times 5^3)+(1\times 5^2)+(2\times 5)+(4\times 1)$
(f) $(4\times 5^3)+(0\times 5^2)+(4\times 5)+(1\times 1)$
(g) $(4\times 5^4)+(3\times 5^3)+(2\times 5^2)+(1\times 5)+(4\times 1)$
(h) $(4\times 5^4)+(0\times 5^3)+(0\times 5^2)+(4\times 5)+(0\times 1)$
(i) $(6\times 7^3)+(0\times 7^2)+(6\times 7)+(2\times 1)$
(j) $(3\times 7^4)+(4\times 7^3)+(6\times 7^2)+(1\times 7)+(6\times 1)$
(k) $(6\times 7^4)+(0\times 7^3)+(0\times 7^2)+(3\times 7)+(3\times 1)$
(l) $(2\times 5^5)+(1\times 5^4)+(3\times 5^3)+(2\times 5^2)+(4\times 5)+(0\times 1)$
(m) $(2\times 5^4)+(0\times 5^3)+(0\times 5^2)+(0\times 5)+(0\times 1)$
(n) $(4\times 7^4)+(0\times 7^3)+(0\times 7^2)+(0\times 7)+(0\times 1)$
(o) $(1\times 5^5)+(4\times 5^4)+(1\times 5^3)+(3\times 5^2)+(2\times 5)+(1\times 1)$
(p) $(2\times 7^4)+(1\times 7^3)+(6\times 7^2)+(3\times 7)+(6\times 1)$

10.

	0	1	2	3	4	5	6
0	0_7	0_7	0_7	0_7	0_7	0_7	0_7
1	0_7	1_7	2_7	3_7	4_7	5_7	6_7
2	0_7	2_7	4_7	6_7	11_7	13_7	15_7
3	0_7	3_7	6_7	12_7	15_7	21_7	24_7
4	0_7	4_7	11_7	15_7	22_7	26_7	33_7
5	0_7	5_7	13_7	21_7	26_7	34_7	42_7
6	0_7	6_7	15_7	24_7	33_7	42_7	51_7

CHAPTER 5

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2. (f) $3\times\frac{2}{3}=2$; $8\times\frac{5}{8}=5$; $16\times\frac{7}{16}=7$; $2\times\frac{3}{2}=3$; $5\times\frac{9}{5}=9$; $10\times\frac{7}{10}=7$; $6\times\frac{6}{6}=6$

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5. (a) 10 (b) 6 (c) 8 (d) 14 (e) 2 (f) 4 (g) 20 (h) 20 (i) 18

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8. (a) $1\frac{1}{2}$ (b) 1 (c) $1\frac{1}{8}$ (d) 2 (e) $3\frac{1}{2}$
(f) 2 (g) 2 (h) $1\frac{1}{4}$ (i) $2\frac{1}{2}$ (j) 2

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14. (a) 1 (b) $31\frac{1}{2}$ (c) $2\frac{5}{8}$ (d) $2\frac{1}{3}$ (e) 78
(f) $1\frac{1}{4}$ (g) 21 (h) 46 (i) $1\frac{3}{32}$ (j) 6

15. 60¢ 16. 434 mi.

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8. (a) $\underline{4}$ (b) $\underline{3}$ (c) $\underline{1}$ (d) $\overline{4}$ (e) $\underline{21}$ (f) $\overline{5}$
(g) $\overline{6}$ (h) $\overline{84}$ (i) $\overline{9}$ (j) $\underline{35}$ (k) $\overline{8}$ (l) $\underline{36}$

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6. (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{2}{5}$ (e) $\frac{1}{4}$ (f) $\frac{3}{7}$ (g) $\frac{2}{3}$ (h) $\frac{3}{5}$
(i) $\frac{3}{4}$ (j) $\frac{3}{4}$ (k) $\frac{11}{12}$ (l) $\frac{11}{12}$ (m) $\frac{3}{5}$ (n) $\frac{3}{4}$ (o) $\frac{2}{3}$ (p) $\frac{9}{17}$
7. (a) $\frac{6}{10}=\frac{9}{15}=\frac{12}{20}$ (b) $\frac{9}{18}=\frac{3}{6}=\frac{1}{2}$ (c) $\frac{15}{45}=\frac{3}{9}$ (d) $\frac{16}{24}=\frac{8}{12}$

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3. (a) $\frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}$ (b) $\frac{9}{10}, \frac{8}{10}, \frac{4}{10}, \frac{3}{10}, \frac{1}{10}$ (c) $\frac{12}{16}, \frac{11}{16}, \frac{10}{16}, \frac{9}{16}, \frac{8}{16}$
(d) $\frac{10}{12}, \frac{9}{12}, \frac{8}{12}, \frac{4}{12}, \frac{1}{12}$ (e) $\frac{19}{20}, \frac{18}{20}, \frac{17}{20}, \frac{16}{20}, \frac{15}{20}$ (f) $\frac{15}{15}, \frac{14}{15}, \frac{9}{15}, \frac{4}{15}, \frac{1}{15}$

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7. (c) $13\frac{1}{2}$ (d) 1 (e) $2\frac{1}{3}$ (f) $2\frac{5}{6}$ (g) 3 (h) 3 (i) $3\frac{2}{9}$ (j) $2\frac{5}{7}$ (k) $2\frac{13}{25}$ (l) $1\frac{2}{13}$ (m) 2
8. (a) $4\frac{1}{9}$ (b) 7 (c) $5\frac{1}{4}$ (d)$8\frac{4}{5}$ (e) $3\frac{3}{20}$ (f) $1\frac{1}{4}$ (g) 10 (h) $1\frac{2}{15}$ (i) $1\frac{3}{7}$ (j) 2 (k) 12 (l) $1\frac{1}{2}$

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11. (a) $\frac{3}{8}, \frac{1}{4}, \frac{1}{8}$ (b) $\frac{9}{10}, \frac{4}{5}, \frac{7}{10}$ (c) $\frac{11}{12}, \frac{5}{6}, \frac{7}{12}$
(d) $\frac{3}{4}, \frac{11}{16}, \frac{9}{16}$ (e) $\frac{9}{20}, \frac{2}{5}, \frac{7}{20}$ (f) $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}$
12. $\frac{10}{15}$; $\frac{2}{3}$
13. (a) $\frac{1}{4}, \frac{1}{3}, \frac{2}{3}$ (b) $\frac{3}{5}, \frac{3}{4}, \frac{4}{5}$ (c) $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}$
(d) $\frac{2}{3}, \frac{7}{10}, \frac{9}{10}$ (e) $\frac{5}{8}, \frac{4}{5}, \frac{7}{8}$ (f) $\frac{1}{3}, \frac{7}{10}, \frac{9}{10}$

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4. (a) $\frac{9}{10}, \frac{11}{12}, \frac{15}{16}$ (b) $\frac{5}{8}, \frac{7}{10}, \frac{3}{4}$ (c) $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}$
(d) $\frac{7}{16}, \frac{1}{2}, \frac{5}{8}$ (e) $\frac{2}{3}, \frac{11}{12}, \frac{14}{15}$ (f) $\frac{13}{16}, \frac{7}{8}, \frac{9}{10}$
5. (a) $\frac{1}{10}$ (b) $\frac{2}{3}$ (c) $\frac{2}{7}$ (d) $\frac{7}{16}$ (e) $\frac{1}{10}$ (f) $\frac{1}{3}$ (g) $\frac{7}{60}$ (h) $\frac{11}{15}$

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2. (a) $\frac{2}{7}$ (b) $1\frac{1}{3}$ (c) $\frac{6}{11}$ (d) $\frac{7}{9}$ (e) 1 (f) $\frac{11}{20}$
(g) $\frac{13}{15}$ (h) $\frac{7}{16}$ (i) $\frac{2}{3}$ (j) $\frac{3}{4}$ (k) $\frac{4}{5}$ (l) $\frac{7}{8}$

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4. (a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{4}{5}$ (d) $4\frac{2}{5}$ (e) $1\frac{7}{12}$ (f) $7\frac{3}{5}$
(g) 2 (h) 6 (i) 21 (j) $4\frac{4}{15}$ (k) $66\frac{2}{3}$ (l) 84
(m) 70 (n) 33 (o) 180 (p) $\frac{13}{16}$

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6. (a) 1 (b) $\frac{5}{8}$ (c) $1\frac{3}{5}$ (d) $2\frac{1}{4}$ (e) $1\frac{1}{3}$ (f) $\frac{3}{16}$ (g) $1\frac{7}{8}$ (h) $5\frac{1}{4}$ (i) $12\frac{3}{4}$ (j) $6\frac{1}{4}$

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3. (b) $2\frac{1}{12}$; $2\frac{1}{12}$ (c) $3\frac{1}{8}$; $3\frac{1}{8}$ (d) $\frac{35}{108}$; $\frac{35}{108}$ (e) $2\frac{1}{3}$; $2\frac{1}{3}$
(f) 4; 4 (g) $6\frac{1}{3}$; $6\frac{1}{3}$ (h) $26\frac{1}{4}$; $26\frac{1}{4}$ (i) $\frac{55}{108}$; $\frac{55}{108}$

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7. (a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{64}$ (d) $\frac{1}{8}$ (e) 1 (f) $\frac{19}{30}$ (g) $\frac{1}{3}$
(h) $\frac{3}{32}$ (i) $\frac{1}{5}$ (j) $\frac{1}{6}$ (k) $\frac{1}{6}$ (l) $1\frac{1}{3}$ (m) $5\frac{5}{8}$ (n) 2
(o) $19\frac{1}{2}$ (p) $1\frac{5}{6}$ (q) 4 (r) 24 (s) 4 (t) $16\frac{1}{2}$ (u) 760
(v) $\frac{112}{153}$ (w) $33\frac{1}{3}$ (x) 85 (y) $\frac{2}{3}$ (z) $3285\frac{5}{7}$
1. $1687\frac{1}{2}$

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2. $81\frac{1}{3}$ 3. $8.91 4. 280 5. $2167\frac{1}{2}$
6. $5\frac{1}{2}$ 7. $\frac{19}{32}$ 8. $1\frac{13}{40}$ 9. $69\frac{23}{117}$

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7. $1\frac{1}{3}$; $4\frac{1}{2}$; $1\frac{1}{4}$; $\frac{10}{11}$; $1\frac{3}{5}$; $1\frac{1}{9}$; $2\frac{2}{7}$; $\frac{5}{21}$; $\frac{2}{7}$; $1\frac{1}{14}$
9. $\frac{3}{10}$; $\frac{4}{19}$; $\frac{10}{29}$; $\frac{9}{49}$

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6. (a) $3\frac{1}{3}$ (b) $\frac{4}{9}$ (c) $\frac{7}{16}$

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(d) $\frac{1}{3}$ (e) $1\frac{1}{3}$ (f) $\frac{15}{32}$ (g) 2 (h) $1\frac{47}{85}$ (i) $2\frac{2}{5}$
(j) 8 (k) 3 (l) $\frac{5}{9}$ (m) $\frac{6}{11}$ (n) $4\frac{2}{11}$ (o) $\frac{5}{9}$
(p) $\frac{2}{3}$ (q) $\frac{27}{38}$ (r) $\frac{7}{60}$ (s) 21 (t) $7\frac{1}{14}$ (u) $\frac{5}{7}$
(v) $3\frac{9}{10}$ (w) 1 (x) $1\frac{2}{9}$ (y) $2\frac{1}{10}$ (z) $\frac{2}{45}$ (aa) $\frac{1}{4}$
1. 50 2. 120 3. $\frac{1}{4}$ 4. 61

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5. $\frac{7}{144}$ 6. $9\frac{1}{4}$ cran 7. 333 8. $21\frac{1}{3}''$ 9. $\frac{2}{9}$ 10. $13\frac{7}{24}$
1. 3 2. 4 3. $\frac{7}{9}$
4. (a) $\frac{5}{8}$ (b) $\frac{1}{2}$ (c) $1\frac{3}{5}$ (d) 2 (e) $1\frac{5}{6}$ (f) 2
(g) $1\frac{3}{5}$ (h) $5\frac{2}{3}$ (i) $3\frac{1}{4}$ (j) 2 (k) $1\frac{1}{6}$ (l) $4\frac{2}{3}$

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8. (a) $1\frac{8}{45}$ (b) $2\frac{9}{28}$ (c) $2\frac{13}{24}$ (d) $2\frac{71}{77}$ (e) $1\frac{27}{40}$
(f) $2\frac{27}{40}$ (g) $1\frac{1}{12}$ (h) $2\frac{5}{22}$ (i) $2\frac{35}{36}$ (j) $1\frac{1}{20}$
(k) $1\frac{13}{84}$ (l) $3\frac{1}{4}$ (m) $\frac{23}{30}$ (n) $1\frac{13}{20}$ (o) $1\frac{1}{2}$
9. (a) $1\frac{1}{6}$ (b) $1\frac{1}{2}$ (c) $5\frac{3}{8}$ (d) $1\frac{36}{55}$ (e) $1\frac{7}{10}$
(f) $2\frac{1}{4}$ (g) $\frac{7}{8}$ (h) $1\frac{41}{60}$ (i) 5

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12. (a) $4\frac{1}{2}$ (b) $1\frac{9}{10}$ (c) $2\frac{5}{8}$ (d) $2\frac{11}{40}$ (e) $1\frac{169}{360}$ (f) $1\frac{19}{36}$
15. (a) $2\frac{5}{12}$ (b) $9\frac{59}{72}$ (c) $5\frac{11}{30}$ (d) 5 (e) $8\frac{47}{120}$ (f) $10\frac{5}{8}$ (g) $13\frac{149}{280}$ (h) $8\frac{2}{9}$

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17. (a) $8\frac{1}{40}$ (b) 10 (c) $9\frac{11}{24}$ (d) $13\frac{9}{20}$ (e) $9\frac{2}{15}$
(f) $8\frac{3}{16}$ (g) $5\frac{5}{24}$ (h) $17\frac{8}{15}$ (i) $5\frac{7}{12}$ (j) $14\frac{35}{72}$
1. $37\frac{9}{10}$ 2. $13\frac{1}{8}$ 3. $25\frac{29}{60}$ 4. $33\frac{2}{3}$ yr.
5. 73

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6. $11.22 7. $20\frac{11}{24}$ 8. no 9. $10\frac{3}{8}$ 10. 1

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3. (b) $3\frac{5}{12}$ (c) $\frac{5}{6}$ (d) 1150 (e) 456 (f) 265 (g) 210 (h) $13\frac{1}{3}$ (i) $1\frac{1}{18}$
2. (a) 1 (b) 1 (c) $\frac{1}{2}$ (d) 1 (e) $\frac{3}{10}$
(f) 0 (g) $1\frac{1}{2}$ (h) $\frac{1}{5}$ (i) $\frac{1}{3}$ (j) $\frac{7}{8}$

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6. (a) $\{1\frac{1}{6}\}$ (b) $\{1\frac{1}{4}\}$ (c) $\{2\frac{1}{10}\}$ (d) $\{\frac{19}{20}\}$ (e) $\{2\frac{7}{24}\}$
(f) $\{2\frac{31}{40}\}$ (g) $\{3\frac{5}{48}\}$ (h) $\{\frac{13}{15}\}$ (i) $\{2\frac{5}{18}\}$ (j) $\{1\frac{119}{120}\}$

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10. (a) $\{\frac{1}{9}\}$ (b) $\{\frac{1}{4}\}$ (c) $\{\frac{3}{4}\}$ (d) 0 (e) $\{\frac{5}{18}\}$
(f) $\{\frac{17}{36}\}$ (g) $\{2\frac{13}{120}\}$ (h) $\{\frac{71}{72}\}$ (i) $\{\frac{11}{60}\}$ (j) $\{\frac{27}{40}\}$

1. $1\frac{5}{8}$ 2. $4'7\frac{15}{16}''$ 3. $3\frac{29}{32}''$ 4. $12\frac{1}{3}$
5. $3\frac{1}{10}$ 6. $22\frac{3}{4}$ 7. $6\frac{5}{6}$ 8. $\frac{14}{15}$

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7. (b) $\frac{7}{24}$ (c) $\frac{11}{16}$ (d) 2 (e) $\frac{147}{160}$ (f) $3\frac{2}{3}$ (g) $6\frac{3}{4}$ (h) $3\frac{19}{70}$ (i) $6\frac{1}{2}$
(j) $1\frac{1}{4}$ (k) $\frac{51}{80}$ (l) $\frac{61}{96}$ (m) $\frac{14}{15}$ (n) $6\frac{5}{12}$ (o) $9\frac{3}{4}$ (p) $6\frac{1}{4}$ (q) $\frac{46}{63}$

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8. (b) $\{\frac{1}{2}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{2}{2}\}$ (c) $\{\frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}, \frac{7}{5}\}$ (d) $\{\frac{1}{4}, \frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{3}{8}\}$
(e) $\{\frac{2}{7}, \frac{5}{14}, \frac{6}{14}, \frac{7}{14}, \frac{9}{14}\}$ (f) $\{\frac{5}{8}, \frac{26}{40}, \frac{27}{40}, \frac{28}{40}, \frac{9}{10}\}$ (g) $\{\frac{2}{3}, \frac{17}{24}, \frac{18}{24}, \frac{19}{24}, \frac{5}{6}\}$
(h) $\{\frac{7}{2}, \frac{29}{8}, \frac{30}{8}, \frac{31}{8}, \frac{8}{2}\}$ (i) $\{\frac{4}{3}, \frac{81}{60}, \frac{82}{60}, \frac{83}{60}, \frac{7}{5}\}$ (j) $\{\frac{11}{2}, \frac{23}{4}, \frac{24}{4}, \frac{25}{4}, \frac{27}{4}\}$
(k) $\{\frac{7}{15}, \frac{22}{45}, \frac{23}{45}, \frac{24}{45}, \frac{7}{9}\}$ (l) $\{\frac{1}{1}, \frac{9}{8}, \frac{10}{8}, \frac{11}{8}, \frac{2}{1}\}$ (m) $\{\frac{3}{1}, \frac{49}{16}, \frac{50}{16}, \frac{51}{16}, \frac{7}{2}\}$

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3. (a) $\{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}\}$ (b) $\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\}$ (c) $\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\}$
(d) $\{\frac{1}{6}, \frac{2}{6}, \frac{4}{6}, \frac{7}{6}\}$ (e) $\{\frac{3}{8}, \frac{10}{8}, \frac{11}{8}, \frac{12}{8}\}$ (f) $\{\frac{4}{5}, \frac{11}{5}, \frac{14}{5}, \frac{20}{5}\}$
(g) $\{\frac{1}{2}, \frac{5}{3}, \frac{7}{2}, \frac{13}{2}\}$ (h) $\{\frac{7}{10}, \frac{11}{10}, \frac{14}{10}, \frac{20}{10}\}$ (i) $\{\frac{3}{3}, \frac{4}{3}, \frac{9}{3}, \frac{13}{3}\}$
(j) $\{\frac{5}{4}, \frac{7}{4}, \frac{8}{4}, \frac{13}{4}\}$ (k) $\{\frac{1}{8}, \frac{1}{4}, \frac{1}{2}\}$ (l) $\{\frac{1}{2}, \frac{2}{3}, \frac{5}{6}\}$
(m) $\{\frac{1}{10}, \frac{7}{10}, \frac{4}{5}\}$ (n) $\{\frac{3}{4}, \frac{5}{6}, \frac{11}{12}\}$ (o) $\{\frac{1}{5}, \frac{14}{10}, \frac{3}{2}\}$
(p) $\{\frac{5}{8}, \frac{11}{16}, \frac{9}{4}\}$ (q) $\{\frac{2}{3}, \frac{7}{4}, \frac{5}{2}\}$ (r) $\{\frac{4}{9}, \frac{8}{9}, \frac{11}{3}\}$
(s) $\{\frac{5}{8}, \frac{5}{4}, \frac{5}{2}\}$ (t) $\{\frac{6}{5}, \frac{4}{3}\}$

4. (a) $\{\frac{1}{4}, \frac{2}{4} \cdots \frac{7}{4}, \frac{8}{4}\}$ (b) $\{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}, \frac{7}{3}, \frac{8}{3}, \frac{9}{3}\}$ (c) $\{\frac{3}{4}, \frac{3}{2}, 2, \frac{11}{4}, \frac{7}{2}\}$
(d) $\{\frac{1}{3}, \frac{1}{2}, 1, \frac{7}{6}, \frac{5}{3}, 2\}$ (e) $\{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \frac{11}{12}\}$

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10. (a) $\frac{1}{3} < x < \frac{5}{6}$ (b) $\frac{2}{5} < x$ (c) $\frac{1}{3} < x < \frac{5}{3}$ (d) $\frac{2}{2} < x$ (e) $\frac{5}{5} < x < \frac{10}{5}$

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1. $\frac{7}{30}$ 2. $3\frac{1}{3}$ 3. $\frac{3}{10}$ 4. 78 5. $11\frac{3}{17}$ 6. $18\frac{1}{3}$

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7. $2\frac{1}{2}$ 8. 65 9. 9 10. 2 11. \$5833.33
12. $\frac{1}{90}$ 13. $\frac{1}{4}$ 14. 9 15. $5\frac{1}{16}''$ 16. $1206\frac{2}{3}$
17. $\frac{17}{120}$ 18. $1\frac{1}{2}$ 19. $10\frac{13}{16}$ 20. $2\frac{7}{24}$ 21. $1\frac{1}{4}$

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22. $35\frac{7}{8}$ 23. $6\frac{1}{2}$ 24. 36 25. 82 26. $4\frac{21}{32}$
27. $7\frac{3}{4}$ 28. $11\frac{7}{12}$ 29. $16\frac{37}{48}$ 30. $18\frac{14}{15}$

CHAPTER TEST

1. (a) $\frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ (b) $\frac{4}{48}, \frac{5}{60}, \frac{6}{72}$ (c) $\frac{4}{80}, \frac{5}{100}, \frac{6}{120}$
(d) $\frac{12}{32}, \frac{15}{40}, \frac{18}{48}$ (e) $\frac{16}{20}, \frac{20}{25}, \frac{24}{30}$ (f) $\frac{28}{64}, \frac{35}{80}, \frac{42}{96}$
(g) $\frac{16}{60}, \frac{20}{75}, \frac{24}{90}$ (h) $\frac{36}{40}, \frac{45}{50}, \frac{54}{60}$ (i) $\frac{8}{28}, \frac{10}{35}, \frac{12}{42}$

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2. (a) $\frac{}{30}$ (b) $\frac{3}{}$ (c) $\frac{3}{}$ (d) $\frac{12}{}$ (e) $\frac{}{36}$ (f) $\frac{9}{}$ (g) $\frac{}{144}$ (h) $\frac{20}{}$
3. (a) $\frac{1}{4}$ (b) $\frac{3}{7}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$ (e) $\frac{11}{13}$ (f) $\frac{87}{124}$
5. (a) 9 (b) $3\frac{3}{7}$ (c) $2\frac{1}{13}$ (d) 4 (e) $6\frac{2}{3}$ (f) $5\frac{1}{3}$
6. (a) $\frac{1}{5}$, $\frac{7}{10}$, $\frac{4}{5}$ (b) $\frac{9}{24}$, $\frac{2}{3}$, $\frac{5}{6}$ (c) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$
(d) $\frac{5}{9}$, $\frac{3}{4}$, $\frac{11}{12}$ (e) $\frac{3}{10}$, $\frac{3}{5}$, $\frac{3}{4}$ (f) $\frac{5}{20}$, $\frac{4}{15}$, $\frac{3}{10}$
7. (a) $2\frac{5}{12}$ (b) $1\frac{13}{15}$ (c) $2\frac{1}{20}$ (d) $1\frac{17}{50}$ (e) $1\frac{73}{144}$ (f) $2\frac{17}{30}$
(g) $1\frac{47}{48}$ (h) $1\frac{5}{9}$ (i) $1\frac{31}{60}$ (j) $2\frac{1}{15}$ (k) $5\frac{5}{6}$ (l) $19\frac{13}{60}$
(m) $13\frac{2}{3}$ (n) $10\frac{47}{120}$ (o) $12\frac{13}{120}$ (p) $10\frac{79}{144}$ (q) $7\frac{29}{90}$ (r) $11\frac{107}{120}$
8. (a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{43}{45}$ (d) $3\frac{24}{29}$ (e) $4\frac{19}{24}$ (f) $5\frac{1}{10}$
(g) $1\frac{1}{10}$ (h) $6\frac{53}{60}$ (i) $\frac{17}{24}$ (j) $3\frac{41}{72}$ (k) $\frac{40}{91}$ (l) $1\frac{63}{80}$
9. (a) $1\frac{3}{5}$ (b) $2\frac{2}{5}$ (c) $1\frac{1}{2}$ (d) $12\frac{4}{7}$ (e) $\frac{1}{8}$ (f) $3\frac{2}{3}$
(g) 16 (h) $\frac{1}{5}$ (i) $3\frac{13}{36}$ (j) $5\frac{1}{2}$
10. (a) $1\frac{1}{2}$ (b) $2\frac{2}{3}$ (c) 5 (d) $\frac{1}{5}$ (e) $\frac{2}{3}$ (f) 3 (g) $\frac{1}{12}$ (h) $\frac{3}{8}$

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11. (a) 6 (b) 6 (c) $4\frac{1}{2}$ (d) 3 (e) $2\frac{3}{4}$ (f) $2\frac{1}{4}$
(g) $2\frac{1}{10}$ (h) $1\frac{32}{33}$ (i) 24 (j) $\frac{315}{512}$ (k) $\frac{1}{4}$ (l) $3\frac{1}{2}$
(m) $\frac{3}{8}$ (n) $1\frac{61}{93}$ (o) $1\frac{3}{8}$ (p) $\frac{43}{96}$ (q) $\frac{3}{7}$ (r) 45
12. 5′ 13. $\frac{7}{8}$″ 14. $3\frac{41}{72}$ 15. 480 mi. 16. 486 mi.
17. $7\frac{1}{2}$ mi. 18. $3\frac{1}{3}$ 19. $2\frac{19}{96}$′ 20. $6\frac{1}{16}$″

CHAPTER 6

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2. 1
10
100

3. 100
10
1

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4. tenths, $\frac{1}{10}$, 456.9
5. $\frac{1}{10}$ of 1 tenth; $\frac{1}{10}$ of 1 hundredth; $\frac{1}{10}$ of 1 thousandth

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7. (b) $(3\times10)+(5\times1)+\left(0\times\frac{1}{10}\right)+\left(5\times\frac{1}{10^2}\right)+\left(7\times\frac{1}{10^3}\right)+\left(2\times\frac{1}{10^4}\right)$

(c) $(9\times1)+\left(6\times\frac{1}{10}\right)+\left(0\times\frac{1}{10^2}\right)+\left(0\times\frac{1}{10^3}\right)+\left(5\times\frac{1}{10^4}\right)$

(d) $(0\times1)+\left(3\times\frac{1}{10}\right)+\left(2\times\frac{1}{10^2}\right)+\left(8\times\frac{1}{10^3}\right)+\left(7\times\frac{1}{10^4}\right)$

8. (b) $(4\times1)+\frac{3}{10}+\frac{6}{10^2}+\frac{7}{10^3}$ (c) $(1\times10)+(2\times1)+\frac{9}{10}+\frac{8}{10^2}$

(d) $(5\times1)+\frac{4}{10}+\frac{0}{10^2}+\frac{7}{10^3}+\frac{1}{10^4}$ (e) $(2\times10)+(7\times1)+\frac{2}{10}+\frac{7}{10^2}$

(f) $0+\frac{1}{10}+\frac{6}{10^2}+\frac{5}{10^3}+\frac{0}{10^4}$

(g) $(1\times10)+(2\times1)+\frac{3}{10}+\frac{0}{10^2}+\frac{4}{10^3}+\frac{1}{10^4}$

(h) $0+\frac{0}{10}+\frac{0}{10^2}+\frac{0}{10^3}+\frac{1}{10^4}$

(i) $0+\frac{1}{10}+\frac{0}{10^2}+\frac{5}{10^3}$

(j) $0+\frac{0}{10}+\frac{1}{10^2}+\frac{0}{10^3}+\frac{5}{10^4}$

(k) $(1\times1)+\frac{0}{10}+\frac{5}{10^2}$

(l) $(2\times1)+\frac{6}{10}+\frac{7}{10^2}+\frac{5}{10^3}$

(m) $(4\times10)+(1\times1)+\frac{9}{10}+\frac{8}{10^2}+\frac{3}{10^3}$

(n) $(6\times10^2)+(2\times10)+(0\times1)+\frac{1}{10}$

(o) $(5\times1)+\frac{3}{10}+\frac{4}{10^2}+\frac{7}{10^3}+\frac{7}{10^4}$

(p) $(5\times1)+\frac{0}{10}+\frac{9}{10^2}+\frac{4}{10^3}+\frac{1}{10^4}$

(q) $(6\times1)+\frac{0}{10}+\frac{0}{10^2}+\frac{0}{10^3}+\frac{0}{10^4}$

(r) $(7\times1)+\frac{8}{10}+\frac{8}{10^2}+\frac{1}{10^3}+\frac{8}{10^4}$

(s) $(8\times1)+\frac{0}{10}+\frac{8}{10^2}+\frac{0}{10^3}+\frac{8}{10^4}$

(t) $(1\times10)+(9\times1)+\frac{6}{10}+\frac{8}{10^2}+\frac{4}{10^3}$

(u) $(2\times10)+(3\times1)+\frac{6}{10}+\frac{0}{10^2}+\frac{8}{10^3}+\frac{3}{10^4}$

9. (a) $(6\times1)+\frac{9}{10}+\frac{4}{10^2}=6\frac{94}{100}$

(b) $(1\times10)+(7\times1)+\frac{8}{10}=17\frac{8}{10}$

(c) $(1\times1)+\frac{0}{10}+\frac{4}{10^2}=1\frac{4}{100}$

(d) $(5\times1)+\frac{0}{10}+\frac{0}{10^2}+\frac{8}{10^3}=5\frac{8}{1000}$

(e) $(7\times1)+\frac{0}{10}+\frac{1}{10^2}+\frac{8}{10^3}=7\frac{18}{1000}$

(f) $(6\times1)+\frac{2}{10}+\frac{0}{10^2}+\frac{4}{10^3}=6\frac{204}{1000}$

(g) $(8\times1)+\frac{1}{10}+\frac{0}{10^2}+\frac{9}{10^3}=8\frac{109}{1000}$

(h) $(1\times10)+(9\times1)+\frac{6}{10}+\frac{1}{10^2}+\frac{4}{10^3}+\frac{7}{10^4}=19\frac{6147}{10000}$

(i) $(5\times1)+\frac{4}{10}+\frac{0}{10^2}+\frac{1}{10^3}+\frac{4}{10^4}=5\frac{4014}{10{,}000}$

(j) $(1\times1)+\frac{3}{10}+\frac{3}{10^2}+\frac{0}{10^3}+\frac{6}{10^4}=1\frac{3306}{10000}$

(k) $(2\times1)+\frac{0}{10}+\frac{1}{10^2}+\frac{0}{10^3}+\frac{4}{10^4}=2\frac{104}{10000}$

(l) $(5\times1)+\frac{0}{10}+\frac{0}{10^2}+\frac{0}{10^3}+\frac{6}{10^4}=5\frac{6}{10000}$

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11. (b) $8\frac{46}{100}$ (c) $23\frac{48}{1000}$ (d) $9\frac{649}{1000}$ (e) $306\frac{8}{10}$
(f) $1000\frac{6042}{10000}$ (g) $7\frac{415}{10000}$ (h) $\frac{14}{100000}$ (i) $96\frac{405}{1000}$
(j) $\frac{105}{10000}$

13. (a) 4.15 (b) 6.5 (c) .014 (d) 3.685 (e) .0005 (f) 465.9
(g) 73.149 (h) 6.94506

14. (a) .47 (b) 43.9 (c) .084 (d) 706.53 (e) 3009 (f) 403.082
(g) 500.005 (h) 34.0701

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17. (b) .125 (c) .55 (d) .375 (e) .325 (f) .28 (g) .5625 (h) .38
(i) .3375 (j) .9375 (k) .62 (l) .6875 (m) .15 (n) .625 (o) .16
(p) .15625

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3. (b) {.1, .5, .7} (c) {.0, .8, .9, 1.0} (d) {.1, .2, 1.0, 2.0}
(e) {.6, .7, .8, .9} (f) {.1, .6, .7, 1.02} (g) {.4, .9, 1.4, 1.9}

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7. (a) {.062, .065, .069} (b) {1.50, 1.55, 1.60} (c) {.999, 1.000, 1.001}
(d) {.08, .10, .11} (e) {3.009, 3.010, 3.014, 3.016}

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2. (a) 1.3 (b) 4.46 (c) 13.902 (d) 2.074 (e) 6.639 (f) 15.354
(g) 14.045 (h) 12.393 (i) 108.9009

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6. (a) 3.24 (b) 3.595 (c) 1.718 (d) 10.896 (e) .098 (f) 1.681
(g) 2.165 (h) 3.86 (i) .449 (j) 16.0879 (k) 15.9989 (l) 8.7486
(m) 1.6185 (n) 4.9276 (o) 2.8937

1. 1663.14

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2. .55 in. 3. .119 4. 22.97 5. 63.8 6. 10.6 7. $179.27 8. 811
9. $3.97 10. $.58

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5. (a) 4.06 (b) 15.12 (c) 75.6 (d) 18.744 (e) 99.28 (f) 2.132
(g) .5434 (h) 1.04 (i) 83.6 (j) 136.77 (k) 21.805 (l) 1.664

6. (a) .006 (b) .003 (c) .0042 (d) .0552 (e) .0084 (f) .0064
(g) .0012 (h) .01008 (i) .0618 (j) .00001 (k) .00312 (l) .0784
(m) .02625 (n) 1.224 (o) .0108 (p) .0378 (q) .07881 (r) .0296
(s) .6732 (t) .08201 (u) .06919 (v) .2163 (w) .42107

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8. (b) 16.32 (c) 6.97 (d) 120.8 (e) 262.5 (f) 13.137
(g) .0087 (h) 74.74038 (i) 13.6112

10. (a) 14.22 (b) 26.3576 (c) 2.124 (d) .054 (e) 6.6625
(f) 80.86715 (g) 167.914 (h) 98.01

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12. (a) 4 (b) 31 (c) 943 (d) 5060 (e) 174 (f) 14001
(g) 420 (h) 60 (i) 350 (j) .5 (k) 1

13. (a) 1230 (b) 32.4 (c) 1892 (d) 122.4 (e) 1330.7 (f) 72
(g) 11200 (h) 64 (i) 90 (j) 561.12 (k) 1337 (l) 9540

14. (a) .6 (b) 1.7 (c) 2.6 (d) .84 (e) 2.73 (f) 1.595
(g) .6842 (h) .06 (i) .006 (j) .01 (k) .0009 (l) .2015

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15. (a) 7.2 (b) 2.512 (c) 6.2727 (d) 7.52 (e) 1.434 (f) .0286

16. (a) .07 (b) .16 (c) .1 (d) .012 (e) .004 (f) .008
(g) .024 (h) .0914 (i) .01004 (j) .0015 (k) .0024 (l) .0006
(m) .0104 (n) 4.368 (o) .00468 (p) .00308 (q) .01732 (r) 1
(s) .006958 (t) 4.93 (u) .00065

17. (b) 2.92 (c) 12.7512 (d) 2.984 (e) 1.288 (f) 24 (g) .21426

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3. (a) 33.5 (b) 44.875 (c) 23.16 (d) 31.55 (e) 12.625 (f) 7.4

(g) 38.05 (h) 142.9375

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7. (a) .003 (b) 20 (c) 2 (d) 3 (e) 130 (f) .7

(g) 250 (h) 204.9 (i) .01 (j) .3 (k) 4 (l) 800

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10. (a) .8 (b) .375 (c) .75 (d) .45 (e) .625 (f) .325

(g) .6875 (h) .4375 (i) .0375 (j) .2125 (k) .025 (l) .3125

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3. (a) 2.18 (b) 2.05 (c) 104.1 (d) .00842 (e) 21.3 (f) 38100

1. 36.14 2. .1125 3. 1.86 4. 24 5. 104

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6. 16 7. (a) 16.21 (b) 21.01 (c) 15.55

8. (a) 9 (b) (i) 39.75 (ii) 28.0 (iii) 19.875

9. 24 10. $9.72 11. $3.08 12. .20 13. $1171.10 14. $157.06

15. 960 16. 67014″ 17. 23.32″ 18. $14\frac{2}{3}$ 19. (a) 279 (b) 7.25

20. $8.93 21. 58.95″ (22) 649.75 23. 317.975 24. $54.48 25. 9.75

26. .9998 27. 5000 28. 70 29. 7.7982 30. 4.63

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6. (a) $.\dot{1}$ (b) $.1\dot{6}$ (c) $.2\dot{7}$ (d) $.\dot{6}$ (e) $.08\dot{3}$ (f) $.58\dot{3}$ (g) $.\dot{4}$ (h) $.41\dot{6}$

(i) $.6\dot{1}$ (j) .5

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2. (a) 7 (b) 9 (c) 4 (d) 18 (e) 3.2 (f) .8 (g) 12.4

(h) 30.5 (i) 5.29 (j) 10.63 (k) 35.67 (l) .01

4. (a) 84 (b) 90 (c) 301.6 (d) 90.8 (e) 76 (f) 316.7 (g) .9

(h) 1430 (i) 580 (j) 3.2 (k) 1510 (l) 65 (m) 1970 (n) 1462.1

(o) .4 (p) 42

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7. (a) .27 (b) 1.59 (c) .38 (d) .45 (e) .135 (f) .426 (g) .057

(h) .088 (i) 1.2046 (j) .0964 (k) .0018 (l) .0009

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11. (a) 1.37 (b) .137 (c) .0137 (d) .69 (e) .069 (f) .0069 (g) .07

(h) .007 (i) .0007 (j) .004 (k) .0004 (l) .00004 (m) .1205

(n) .01205 (o) .001205 (p) .0003 (q) .00003

(r) .000003

PAGE 204

5. 3.1×10^3; 3.6×10^7 7.85×10^3; 6.7×10^7 7.93×10^3; 9.3×10^7

4.27×10^3; 1.42×10^8 8.93×10^4 4.83×10^8 7.5×10^4 8.86×10^8

3.32×10^4 1.78×10^9 3.09×10^4 2.795×10^9 3.6×10^3 3.7×10^9

PAGE 204

6. (a) 65000 (b) 709000 (c) 5240 (d) 665 (e) 8,040,000

(f) 11,100 (g) 3,320,000 (h) 29,500,000

7. 6.696×10^8

8. 1.598905×10^6 miles per day 6.6621×10^4 miles per hour
Based on $365\frac{1}{4}$ days.

PAGE 207

5. (a) .1 (b) .9 (c) 12.3 (d) 1.0 (e) .9 (f) .3 (g) 12.0 (h) .2
(i) 2.1 (j) 2.0 (k) .1 (l) 12.0 (m) 12.7 (n) 13.9 (o) .2 (p) .7

6. (a) 11.98 (b) 13.65 (c) .47 (d) .67 (e) .10 (f) 5.01 (g) .00 (h) .33
(i) .18 (j) .80 (k) 1.41 (l) 2.72 (m) .00 (n) 1.00 (o) 3.96 (p) .90

7. (a) 4.341 (b) .082 (c) .333 (d) .667 (e) .815 (f) 3.026
(g) 14.617 (h) 9.858 (i) 4.000 (j) .099 (k) .142 (l) .001
(m) .000 (n) .000 (o) .091 (p) .624

PAGE 208

9. (a) .79376 (b) .171 (c) 10.6495 (d) .0025088 (e) .0901
(f) .069012 (g) .70902 (h) .71824 (i) .0243824 (j) .007395

11. (a) 30 (b) .031 (c) 80 (d) .038 (e) .32 (f) 3.7 (g) 5.86 (h) 1.32

12. (a) 1.2 (b) 1.1 (c) 2.2 (d) .9

PAGE 209

(i) 6.9 (ii) 109.4 (iii) 17.6 (iv) 8.7

CHAPTER TEST

2. (a) $\frac{16}{10}$ (b) $\frac{304}{100}$ (c) $\frac{19}{1000}$ (d) $\frac{515}{100}$ (e) $\frac{1}{1000}$ (f) $\frac{145}{10000}$
(g) $\frac{7605}{1000}$ (h) $\frac{1313}{100}$

PAGE 209

3. (a) .4 (b) .7 (c) .25 (d) .5 (e) .875 (f) .125 (g) .75
(h) .6 (i) .2125 (j) .725 (k) .6875 (l) .26

4. (a) 55.720 (b) 266.9689 (c) 30.8842 (d) 55.0889 (e) 55.9799
(f) 173.826 (g) 48.14966 (h) 1079.917

PAGE 210

5. (a) 7.09 (b) .09 (c) 15.88 (d) 10.18 (e) 5.688 (f) 2.687
(g) 17.049 (h) 18.034 (i) 3.875 (j) 2.328 (k) 9.192 (l) 3.9991

6. (a) 4.68 (b) 119.52 (c) 15.048 (d) 24.6648 (e) 191.952 (f) .1464
(g) .36 (h) .00774 (i) .37845 (j) .03328 (k) 90.25 (l) 151.9542

7. (a) 4 (b) 89.55 (c) .5 (d) 14.23 (e) 30.6 (f) .635
(g) 10.4 (h) 2.05 (i) 81 (j) 379 (k) 10.8 (l) 9.78

8. (a) $.\dot{4}$ (b) $.8\dot{3}$ (c) 1.142857142 . . . (d) $1.08\dot{3}$ (e) $1.\dot{0}\dot{9}$
(f) .307692307 . . . (g) $.458\dot{3}$ (h) $1.2\dot{7}$ (i) $.52\dot{7}$ (j) .020875

9. .26″

PAGE 211

10. (a) 14.5; 145; 1450 (b) .45; 4.5; 45 (c) 34; 340; 3400
(d) 29.86; 298.6; 2986 (e) .006; .06; .6 (f) 20.49; 204.9; 2049
(g) 7.73; 77.3; 773 (h) 392.7; 3927; 39270 (i) 141; 1410; 14100
(j) 130.2; 1302; 13020

11. (a) 4.69; .469; .0469 (b) .204; .0204; .00204 (c) .0069; .00069; .000069
(d) .33; .033; .0033 (e) 21.485; 2.1485; .21485 (f) .901; .0901; .00901
(g) .3; .03; .003 (h) .2754; .02754; .002754 (i) 1.36; .136; .0136
(j) .004; .0004; .00004

13. (a) 1.0×10^7 (b) 4.6×10^4 (c) 2.179×10^5 (d) 5.354×10^6
(e) 6.9×10^6

14. (a) 6500 (b) 30900 (c) 6,000,000 (d) 209,000 (e) 12,150,000

15. 2.64 16. 8.84 lb. 17. .015 18. 15

PAGE 212

19. 17 doz. 20. 3.6 hr. 21. 20 22. .00026 in.

CHAPTER 7

PAGE 213

2. (a) 5¢; 15¢; 20¢ (b) yes

PAGE 214

2. (a) yes (b) equivalent

6. (a) yes

PAGE 215

8. (4, 20), (6, 30), (8, 40), (15, 75), (100, 500)

9. (a) $\{\frac{4}{15}, \frac{8}{30}, \frac{12}{45}, \frac{16}{60}, \ldots\}$ (b) $\{\frac{4}{1}, \frac{8}{2}, \frac{12}{3}, \frac{16}{4} \ldots\}$
(c) $\{\frac{45}{1}, \frac{90}{2}, \frac{135}{3}, \frac{180}{4} \ldots\}$ (d) $\{\frac{2}{3}, \frac{4}{74}, \frac{6}{111}, \frac{8}{148}, \ldots\}$
(e) $\{\frac{650}{1}, \frac{1300}{2}, \frac{1950}{3}, \frac{2600}{4} \ldots\}$

10. (a) {(4, 15), (8, 30), (16, 60) . . .} (b) {(4, 1), (8, 2), (12, 3) . . .}
(c) {(45, 1), (90, 2), (135, 3) . . .} (d) {(2, 37), (4, 74), (8, 111) . . .}
(e) {(650, 1), (1300, 2), (1950, 3) . . .}

PAGE 217

17. a; c; d; e; f; g

PAGE 218

18. c; d; f; h; i

PAGE 219

23. (a) 5 (b) $7\frac{1}{2}$ (c) 60 (d) 19 (e) 20 (f) $1\frac{1}{4}$ (g) 12 (h) 20
(i) 35

25. (a) 6 (b) 36 (c) 16 (d) 12 (e) 30 (f) 24 (g) 49 (h) 256
(i) 42

PAGE 220

5. $\frac{2}{3}$; 2:3; (2, 3)

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10. (a) $\frac{1}{5}$; 1:5; (1, 5) $\frac{1}{8}$; 1:8; (1, 8) $\frac{1}{4}$; 1:4; (1, 4) $\frac{1}{7}$; 1:7; (1, 7)
$\frac{1}{2}$; 1:2; (1, 2) $\frac{1}{4}$; 1:4; (1, 4) $\frac{3}{10}$; 3:10; (3, 10) $\frac{3}{20}$; 3:20; (3,20)

PAGE 222

13. $\frac{5}{6}$ 14. $\frac{8}{5}$

15. (a) 1:3 (b) 1:1 (c) 4:1 (d) 3:2 (e) 5:7 (f) 1:2 (g) 1:2 (h) 5:9
(i) 1:7 (j) 5:7

16. (a) 2:3 (b) 1:3 (c) 1:2 (d) 2:6:3

PAGE 225

8. 18 9. 24 10. 59 11. 8 12. 5235 13. 28 14. 22

PAGE 226

15. 135′ 16. \$150.00 17. \$1.50 18. 22$\frac{1}{2}$′ 19. \$15.00 20. \$25.00

21. 20 22. 4 ft. 23. 154 24. \$609.33 25. \$77.00

PAGE 228

7. 44%; 79%; 10%; 1%; 99%; 68%

8. $\frac{5}{100}$; $\frac{75}{100}$; $\frac{6}{100}$; $\frac{84}{100}$; $\frac{95}{100}$; $\frac{18}{100}$

PAGE 229

2. 100; 60; 40; 100; 75; 75

4. 100%; 135%; 197%; 150%; 250%; 200%; 640%; 750%; 150%; 300%; 175%; 175%; 120%; 200%; 160%; 750%

5. 1$\frac{3}{20}$, 10; 6; 7$\frac{1}{4}$; 1$\frac{3}{4}$; 5$\frac{2}{5}$; 7$\frac{3}{5}$; 5$\frac{11}{20}$; 1$\frac{1}{10}$; 4$\frac{3}{4}$; 25 ; 8

PAGE 231

8. 10—88%; 11—133$\frac{1}{3}$%; 12—100%; 13—128$\frac{4}{7}$%; 14—90%; 15—90%

PAGE 232

6. 1%; 4%; 5%; 28%; 744%; 60%; 80%; 10%; 170%; 130%; 460%; 24$\frac{1}{2}$%; 869$\frac{1}{4}$%; 773$\frac{1}{2}$%; 508—15%; 910.05%
59%; 93%; 139%; 150%; 90%; 30%; 120%; 180%; 375%; 72.75%; 96.25%; 66.25%; 766.45%; 1220.95%; 761.05%; .05%

PAGE 233

7. .05; .84; .30; 1.25; 1.10; .375; .0834; .53; 1.19; .405; .1225; 3.52875; .16 .0482

9. $\frac{1}{3}$; $\frac{3}{8}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{5}{6}$; $\frac{1}{16}$; $\frac{1}{12}$; $\frac{1}{8}$; $\frac{5}{8}$; $\frac{13}{16}$

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12. 25%; 40%; 83$\frac{1}{3}$%; 37$\frac{1}{2}$%; 16$\frac{2}{3}$%; 9%; 12%; 15%; 33$\frac{1}{3}$%; 57$\frac{1}{7}$%; 37$\frac{1}{2}$%; 27$\frac{7}{9}$%; 44$\frac{4}{9}$%; 72$\frac{7}{11}$%; 46$\frac{2}{3}$%; 85$\frac{5}{7}$%; 120%; 266$\frac{2}{3}$%; 316$\frac{2}{3}$%; 411$\frac{1}{9}$%; 587$\frac{1}{2}$%; 291$\frac{2}{3}$%; 1441$\frac{2}{3}$%; 1973$\frac{1}{3}$%; 80%; 70%; 28%; 55%; 12$\frac{1}{2}$%; 5%; 775%; 337$\frac{1}{27}$%

PAGE 235

5. 17 6. Food—\$1700; Rent—\$1360; Clothing—\$1020.00; Savings \$680.00 Other—\$2040.00

7. 70%; \$50.00 8. 750 9. \$2650.00

PAGE 236

10. \$990.00 11. (a) 50 (b) 316$\frac{2}{3}$ (c) 60 (d) 9

12. Copper 1120, Zinc 720, Tin 160 13. (a) \$60.00 (b) \$15.00

14. \$54.00 15. 645 16. 16 17. 18 million 18. 80

19. 300 20. 448 21. \$5200.00 22. 16$\frac{2}{3}$%

PAGE 239

6. \$35.26

7. Refrigerator —Discount \$73.50—Net price \$171.50
Stove —Discount \$43.75—Net price \$131.25
Floor Polisher—Discount \$ 4.00—Net price \$ 35.95
Electric Mixer—Discount \$ 3.73—Net price \$ 21.12

8. \$50.00 9. \$11.40 10. $16\frac{2}{3}$ 11. $16\frac{2}{3}$ 12. $11\frac{1}{9}\%$ 13. $16\frac{2}{3}$

14. \$10.00 15. \$5800.00

PAGE 240

18. \$200 19. $7\frac{1}{2}\%$ 20. \$15500.00 21. $2\frac{1}{2}\%$ 22. \$9471.20

PAGE 241

23. \$800.00 24. \$636.40; \$148.50; \$247.00 \$1002.50 25. \$46.96 26. \$45.50

PAGE 242

31. (a) \$20.00 \$18.75 \$150.00 \$112.50 \$1.25 \$40.50
(b) \$420.00 \$318.75 \$900.00 \$1112.50 \$201.25 \$490.50

32. \$1.50 33. \$3.00; \$15.00; \$1.88; \$117.00; 19¢ ; \$225.00

PAGE 243

35. \$100.00 36. \$200.00; \$700.00; \$550.00; \$400.00; \$500.00; \$3333.33

37. 4 38. $2\frac{1}{2}$ yr; 2 yr; 6 mo.; $1\frac{1}{2}$ yr.; 3 yr.; $1\frac{1}{3}$ yr.

CHAPTER TEST

1. (a) $\frac{6}{1}$; (6, 1) (b) $\frac{4}{1}$; (4, 1) (c) $\frac{49}{12}$; (49, 12)
(d) $\frac{18000}{1}$; (18,000, 1) (e) $\frac{1}{\frac{1}{2}}$; $(1, \frac{1}{2})$ (f) $\frac{29}{1}$; (29, 1)

2. a; b; e; f; h; i; j

3. (a) 3 (b) 2 (c) 35 (d) 26 (e) 4 (f) 63

4. (a) 18 (b) 5 (c) $18\frac{4}{7}$ (d) 12 (e) 10 (f) 20 (g) 56 (h) 27

5. $16\frac{1}{2}$ hr.

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6. \$11.60

7. $\frac{4}{1}$; (4, 1); 4:1 — $\frac{1}{4}$; (1, 4); 1:4
$\frac{1}{2}$; (1, 2); 1:2 — $\frac{2}{1}$; (2, 1); 2:1
$\frac{1}{24}$; (1, 24); 1:24 — $\frac{24}{1}$; (24, 1); 24:1
$\frac{1}{3}$; (1, 3); 1:3 — $\frac{3}{1}$; (3, 1); 3:1
$\frac{3}{8}$; (3, 8); 3:8 — $\frac{8}{3}$; (8, 3); 8:3
$\frac{1}{4}$; (1, 4); 1:4 — $\frac{4}{1}$; (4, 1); 4:1
$\frac{5}{2}$; (5, 2); 5:2 — $\frac{2}{5}$; (2, 5); 2:5
$\frac{256}{1}$; (256, 1); 256:1 — $\frac{1}{256}$; (1, 256); 1:256
$\frac{1}{6}$; (1, 6); 1:6 — $\frac{6}{1}$; (6, 1); 6:1
$\frac{1}{4}$; (1, 4); 1:4 — $\frac{4}{1}$; (4, 1); 4:1

8. 405 9. 38 10. $14\frac{4}{5}$″ 11. 2380 mi.

12. 60 ft. 13. \$6050.00 14. \$22.44

PAGE 246

15. 16%; 27%; 350%; 790%; 81%; 1%; 75%; 20%;
40%; 70%; 50%; 30%; 150%; 380%; 290%; 400%;
317%; 664%

16. $\frac{12}{25}$; $\frac{19}{20}$; $\frac{3}{20}$; $\frac{1}{10}$; $\frac{1}{2}$; $\frac{4}{5}$; $1\frac{7}{20}$; 6; $7\frac{1}{4}$; $2\frac{7}{25}$; 10; 1

17. 4%; 162%; 335.5%; 1420%; 38%; 50%; 450%; 792.6%;
.1%; 156%; 32.5%; 87.5%

18. .1; .35; 1.15; 2.75; .01; .08; .245 .0375;
.061; .09216; 40.01; .00875

19. 50%; 20%; $16\frac{2}{3}$%; $33\frac{1}{3}$%; $12\frac{1}{2}$%; $6\frac{1}{4}$%; $118\frac{3}{4}$%; 280%;
$166\frac{2}{3}$%; $487\frac{1}{2}$%; 530%; $108\frac{1}{3}$%; $441\frac{2}{3}$%; 220%; $588\frac{8}{9}$%; $1283\frac{1}{3}$%;
$1086\frac{2}{3}$%; $461\frac{1}{9}$%

PAGE 246

20. $37\frac{1}{2}$ 21. 60 22. 2000 23. 275 24. $163.76 25. $2187.50

26. (a) 1890 (b) $33\frac{1}{3}$ (c) 30¢ (d) $1\frac{1}{2}$ miles

PAGE 247

27. $136.36 28. $46.40 29. $243.25 30. 87¢

31. $87\frac{1}{2}$ 32. 25 33. 20 34. 15

35. 80¢ 36. 50¢ 37. 6 38. 10

PAGE 248

39. $1160.00 40. b; d; g; h 41. 60 42. $3\frac{1}{2}$

43. 940 44. 4 45. 5

46. 10% store; $3.00 47. 500 48. $645\frac{1}{3}$

PAGE 249

49. 19 50. 36.

CHAPTER 8

PAGE 251

7. B—yes; D—no

PAGE 253

13. (e) $\overline{AX}$ and $\overline{XB}$ (f) yes (g) yes (h) no (i) no

PAGE 255

20. (e) $\overleftrightarrow{XY} = \overleftrightarrow{YX}$; $\overline{LW} = \overline{WL}$; $\overrightarrow{MN} = \overrightarrow{MN}$

PAGE 257

22. (a) X and Y (b) X (c) Y (d) *a* and *b* (e) *c* and *a*

24. Y 25. no

PAGE 258

30. A and C 31. 1 35. A and B

PAGE 262

11. (i) *a, b, c, d, e, f, g* (ii) *c, b, d* (iii) *c, d*

PAGE 263

5. (a) $\overrightarrow{SR}$ and $\overrightarrow{ST}$ (b) S and M (c) ∠RST, ∠TSR, ∠S
$\overrightarrow{ML}$ and $\overrightarrow{MN}$ ∠LMN, ∠NML, ∠M

PAGE 266

9. (a) (i); (iii); (v); (vi); (vii); (viii)
(b) C, D, E, F, G, H (c) LMNO; WXYZ
(d) no (e) 3 (f) 5 (g) equal
(h) $\overline{TV}$; $\overline{ST}$; $\overline{RS}$ (i) 6, hexagon
11. no

PAGE 268

3. (i) $\overline{CD}$ (ii) $\overline{NO}$ (iii) ST (iv) JK

PAGE 269

5. (a) 2 (b) 5 (c) 4 (d) 3 (e) 1
6. (c) (i) 3 (ii) 6 (iii) 8 (iv) 8

PAGE 270

7. AB — 3^+; MN — 4^+; XY — 4^- $3\frac{1}{4}$; $4\frac{1}{4}$; $3\frac{3}{4}$ no

PAGE 272

11. $2\frac{1}{4}$; $2\frac{3}{10}$

PAGE 274

14. A — $3\frac{1}{2}$; $3\frac{1}{2}$; $3\frac{1}{2}$ B — 4; 4; 4
C — 2; $1\frac{3}{4}$; $1\frac{7}{8}$ D — 3; 3; 3
E — 1; 1; $\frac{7}{8}$ F — $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{4}$
G — $4\frac{1}{2}$; $4\frac{1}{2}$; $4\frac{1}{2}$

PAGE 275

4. (i) $4\frac{3}{4}$ (ii) $6\frac{5}{8}$ (iii) $7\frac{3}{8}$ (iv) $7\frac{1}{4}$

PAGE 276

5. 19.6 in. 6. $41\frac{1}{2}$ in. 7. 11″, 9″, 5″
8. $20\frac{2}{5}$″
9. (a) $\overline{AB} = 10'$; $\overline{BC} = 10'$ (b) 92 ft. (c) $16.34
10. 4136 yd. 11. $5.67; 63 ft.

PAGE 279

7. (i) 3 (ii) 2 (iii) 1 10. $3\frac{1}{2}$; $\frac{1}{2}$; $2\frac{1}{2}$

PAGE 282

16. (a) 35° (b) 35° (c) 120° (d) 130° (e) 90°
(f) 112° (g) 85° (h) 53° (i) 150°

PAGE 284

19. (a) 90° — right (b) 138° — obtuse (c) 23° — acute
(d) 180° — straight (e) 112° — obtuse (f) 130° — obtuse
(g) 50° — acute (h) 13° — acute

PAGE 285

21. all

PAGE 288

2. (i) BAD (ii) ABC (iii) BCD (iv) ADC
3. (a) $\overline{LN}$ and $\overline{NM}$ (b) $\overline{AB}$ and $\overline{BC}$ (c) $\overline{GH}$ and $\overline{HC}$
(d) $\overline{SR}$ and $\overline{ST}$ (e) $\overline{ED}$ and $\overline{DC}$ (f) $\overline{CH}$ and $\overline{CD}$

PAGE 290

12. AD and BC

PAGE 291

16. *a*, *b*, *d*

PAGE 292

21. (c) $\overline{ON}$, $\overline{LM}$ (d) $\overline{ZY}$, $\overline{WX}$
$\overline{MN}$, $\overline{LO}$ $\overline{XY}$, $\overline{WZ}$
23. 56 in. 24. 5 25. 10 in., 15 in., 10 in.
27. 92 in. 28. 32 in.

PAGE 293

5. total 180° 8. 180°

PAGE 294

11. 60° = C; X = 90°; X = 20°; T = 20°; D = 155°; P = 60°
12. 107°

PAGE 297

4. right 5. acute 6. no

PAGE 301

16. right 18. $248\frac{44}{47}$ yd. 19. $87\frac{1}{2}$ ft. 20. $87\frac{1}{2}$ yd.

PAGE 302

3. (a) none (b) DEF and ABC (c) GHI

PAGE 303

12. 90° 13. over 90° 14. less than 90° 15. 2 equal angles and 2 equal sides
16. no angles equal, no sides equal 17. all angles equal, all sides equal

PAGE 304

18. (a) right (b) obtuse (c) obtuse (d) right (c) obtuse (f) obtuse
19. (a) isosceles (b) scalene (c) equilateral
(d) isosceles (e) isosceles (f) equilateral
24. yes 25. no 26. yes

PAGE 306

4. (g) (i) radius (ii) diameter (iii) 0
(iv) $\overline{OX}$, $\overline{OM}$, $\overline{OL}$, $\overline{OS}$, $\overline{OY}$, $\overline{ON}$ (v) radius, radius, diameter

PAGE 311

13. (a) $12\frac{4}{7}''$ (b) $18\frac{6}{7}$ ft. (c) $6\frac{2}{7}$ yd. (d) 31.4″
(e) $5\frac{1}{2}''$ (f) 10.99″ (g) 19.7192 yd. (h) $23\frac{1}{21}$
(i) 1183.152 yd. (j) 75.36 ft.
14. (a) $25\frac{1}{7}$ (b) 11 in. (c) $4\frac{5}{7}$ ft. (d) 396 yd.
(e) 62.8 yd. (f) 96.712 yd. (g) 128.74 ft. (h) 110″
15. 219.8 ft. 16. 88″ 17. $113\frac{1}{7}''$
18. 140′ 19. 1.5″

PAGE 314

8. (a) 6 (b) 11 (c) 9 (d) 6 (e) 10 (f) 10
9. (a) 4 (b) 4 (c) 6 (d) 6 (e) 5

PAGE 319

25. 60 sq. ft. or $6\frac{2}{3}$ sq. yd.

PAGE 320

26. (a) 288 sq. in.; 2 sq. ft.; $\frac{2}{9}$ sq. yd. (b) 648 sq. in.; $4\frac{1}{2}$ sq. ft.; $\frac{1}{2}$ sq. yd.
(c) 2592 sq. in.; 18 sq. ft.; 2 sq. yd. (d) 4320 sq. in.; 30 sq. ft.; $3\frac{1}{3}$ sq. yd.
(e) 1458 sq. in.; $10\frac{1}{8}$ sq. ft.; $1\frac{1}{8}$ sq. yd. (f) 90 sq. in.; $\frac{5}{8}$ sq. ft.; $\frac{5}{72}$ sq. yd.
(g) 10 sq. in.; $\frac{5}{72}$ sq. ft.; $\frac{5}{648}$ sq. yd. (h) $273\frac{1}{3}$ sq. in.; $1\frac{97}{108}$ sq. ft.; $\frac{205}{972}$ sq. yd.
28. 1760 29. 3,097,600 30. 4840 31. 27,878,400; 65,340 32. $3226\frac{2}{3}$

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6. A — $2\frac{1}{4}$ sq. in.; B — $1\frac{7}{8}$ sq. in.;
C — $2\frac{13}{32}$ sq. in.; D — $3\frac{3}{4}$ sq. in.

PAGE 323

9. A — $7\frac{1}{2}$ sq. in.; B — 6.48 sq. in.; C — 6.46 sq. ft.;
D — 34.2 sq. in.; E — $71\frac{1}{2}$ sq. ft.; F — 600 sq. ft.

PAGE 327

3. $7\frac{1}{2}$ sq. in.; 9 sq. in.; $3\frac{1}{2}$ sq. in.; $2\frac{1}{2}$ sq. in.; 2.4 sq. in.; $1\frac{7}{8}$ sq. in.

PAGE 330

7. 4 sq. in.; $7\frac{1}{2}$ sq. in.; 7 sq. in.; $4\frac{1}{2}$ sq. in.; $3\frac{1}{2}$ sq. in.; $11\frac{1}{4}$ sq. in.

PAGE 330

2. (a) 3 (b) 720 (c) 12 (d) 63
(e) 4 (f) $302\frac{1}{2}$ (g) 40 (h) 960
(i) 2 (j) 5120 (k) 4840 (l) 3,097,600

PAGE 331

1. (a) triangle; rectangle; parallelogram; square
(b) 24 sq. yd.; 160 sq. ft.; 300 sq. rd.; 121 sq. in.
2. 18 sq. in.; $1\frac{1}{2}$ sq. ft.; $1\frac{1}{2}$ sq. yd.; $5\frac{5}{8}$ sq. ft.; 162 sq. in.; $15\frac{1}{8}$ sq. yd.; 1 acre; $3\frac{3}{4}$ acres; $1\frac{1}{8}$ sq. mi.; $3\frac{3}{4}$ sq. in.; $2\frac{1}{16}$; 45 acres.
3. 400 sq. rd.; 120 sq. ft.; 3 sq. yd.; 96 sq. in.

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4. 24 sq. in.; $1\frac{1}{2}$ sq. ft.; $1\frac{1}{2}$ sq. yd.; $4\frac{1}{4}$ sq. in.; 25 sq. rd.
5. 4 sq. in.; 4 sq. in.; 4 sq. in.; 4 sq. in.; 4 sq. in.
6. 30 sq. in.

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2. $487.50 3. $3\frac{3}{4}$

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4. 12.74 5. $173.25 6. $8.01
7. 392 sq. ft. 8. $200.00 9. $224.40

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10. $152.96 11. 42.05 12. $63.90 13. $61.38

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14. 8 15. $205.00 16. triangle

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8. (a) 3 (b) 8; 12; 6 (c) points (d) line segments (e) parts of planes

PAGE 340

(a) 4; 4 cubic inches (b) 8; 8 cu. in.

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7. (i) 27 cu. in. (ii) 30 cu. in. (iii) 60 cu. in. (iv) 72 cu. in.
8. $19\frac{1}{2}$ cu. in.

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9. $4\frac{1}{2}$ cu. in.
11. (a) 20 cu. in. (b) $31\frac{1}{2}$ cu. in. (c) $22\frac{1}{2}$ cu. in. (d) 32 cu. in.
(e) 20.25 cu. in. (f) 42 cu. in. (g) $202\frac{1}{2}$ cu. in. (h) 220 sq. in.
(i) 113.62 cu. in. (j) $20\frac{2}{5}$ cu. in.

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15. 3888 16. 105 1. 9800 2. 1512 cu. ft. 168 cu. yd.
3. 2250 lb. 4. 80 lb. 5. 4187

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7. A — 4 in.; B — 1 yd.; C — 3 ft.; D — 10″; E — 12″; F — 3 ft.
8. 5,850. 9. $33\frac{1}{3}$ 10. $37\frac{1}{2}$ cu. ft.
11. $93.75 12. $33.33

CHAPTER 9

PAGE 347

1. (a) 19 (b) 32 (c) 96 (d) 93 (e) 24 (f) 88

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2. (a) 92 (b) 31 (c) 22 (d) 90 (e) 29 (f) 88

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5. (a) (i) $8.70 (ii) $0.60 (iii) $4.50 (iv) $8.10 (v) 29:2
   (b) (i) Ontario and Quebec (ii) $\frac{1}{4}$ (iii) 560,000 (iv) 2:1
   (v) One was worked by subtraction, the other by division.

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(d) 1950; 1957; 1963

CHAPTER 10

EXERCISE 1 PAGE 357

1. a, b, c, d, e, f, i, j
2. (a) 6×1, 3+3, etc. (b) 100×1, 50+50, etc.
   (c) 30+5, 7×5, etc. (d) 1×1, 1+0, etc.
   (e) 0×1, 0+0, etc. (f) 5×5, 20+5, etc.
3. (a) 48 (b) 0 (c) 190 (d) 3 (e) 27
   (f) 10 (g) 18 (h) 0 (i) 330 (j) 530
4. (a) 5×10×10+0×10+0×1
   (b) 9×10×10×10+0×10×10+1×10+0×1
   (c) 6×10×10×10+9×10×10+3×10+5×1
   (d) 7×10×10×10+0×10×10+2×10+1×1
   (e) 1×10×10×10×10+0×10×10×10+0×10×10+0×10+0×1
   (f) 9×10×10×10+6×10×10+4×10+9×1
   (g) 1×10×10×10×10×10+0×10×10×10×10+0×10×10×10+0×10×10+0×10+0×1
   (h) 7×10×10×10×10×10+2×10×10×10×10+9×10×10×10+0×10×10+0×10+4×1
   (i) 5×10×10×10×10×10+8×10×10×10×10+9×10×10×10+0×10×10+0×10+0×1
   (j) 1×10×10×10×10×10×10+0×10×10×10×10×10+0×10×10×10×10+0×10×10×10+0×10×10+0×10+0×1
5. (a) 967 (b) 5,010 (c) 700,000 (d) 3210 (e) 6010

EXERCISE 2 PAGE 358

1. (a) $3\times10^2+0\times10+0\times1$ (b) $1\times10^3+0\times10^2+0\times10+0\times1$
(c) $1\times10^4+0\times10^3+0\times10^2+0\times10+0\times1$
(d) $3\times10^2+1\times10+5\times1$
(e) $1\times10^4+0\times10^3+0\times10^2+8\times10+6\times1$
(f) $9\times10^5+0\times10^4+9\times10^3+0\times10^2+9\times10+9\times1$
(g) $1\times10^6+1\times10^5+0\times10^4+0\times10^3+0\times10^2+0\times10+0\times1$
(h) $1\times10^7+0\times10^6+7\times10^5+0\times10^4+8\times10^3+6\times10^2+9\times10+7\times1$

2. (a) 3^4 (b) 10^2 (c) 9^5 (d) 4^4 (e) 50^5 (f) 2^6

3. (a) 729 (b) 16 (c) 64 (d) 1331 (c) 144 (f) 729
(g) 10,000 (h) 10,000 (i) 125 (j) 1296

4. (a) 2 (b) 3 (c) 3 (d)4

5. 3^2, 1

EXERCISE 3 PAGE 358

1. (a) 12
(b) $y=20+19$ (throughout) 39 (c) $100=z+58$; $z=42$
(d) $n=115+86$ 201 (e) $615=287+t$; $t=328$

2. (a) 5
(b) $54=y\times6$ (throughout) 9 (c) $p=7\times5$ 35

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(d) $144=12\times s$ 12 (e) $v=121\div n$ 11

3. (a) (iii) (b) (i), (ii), (iii), (iv)

4. (a) 2294 (b) 5013 (c) 8908 (d) 4266
(e) 9851 (f) 398,703 (g) 817,574 (h) 349,382
(i) 2,502,301 (j) 183,869

EXERCISE 4 PAGE 359

1. (a) {1, 2, 3, 4, 5, 6, 7, 8, 9} (b) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
(c) { } (d) {0}
(e) {10, 11, 12, 13, 14} (f) {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
(g) (i) {1, 5, 25} (ii) {1, 7, 13, 91}
(iii) {1, 2, 3, 4, 6, 9, 12, 18, 36} (iv) {1, 2, 31, 62}
(v) {1, 3, 7, 9, 21, 63} (vi) {1, 41}
(vii) {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}
(h) {16, 24, 32, 40, 48, 56} (i) { } (j) {16, 32, 64}
(k) {11, 22, 33, 44, 55, 66, 77, 88, 99} (l) {2, 4, 6, 12, 18, 36}

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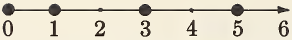
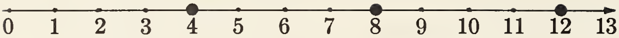
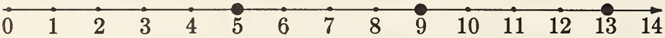
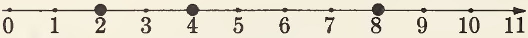
(m) { } (n) {175, 571, 715, 157, 517, 751}
(o) (12, 24, 36, 48, 60, 72, 84, 96} (p) {1, 3, 7, 21}
(q) {1, 3} (r) {1} (s) {30, 60, 90}

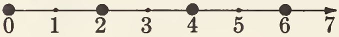
2. (a) {65, 78, 91} (b) {16, 17, 20} (c) {29, 37, 46} (d) {56, 72, 90}
(e) {34, 55, 89} (f) {36, 49, 64} (g) {62, 63, 126} (h) {28, 35, 33}
(i) {121, 242, 249 (j) {29, 31, 37}

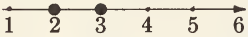
3. (a)


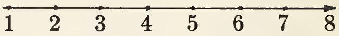
(b)


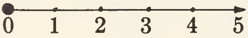
(c)


(d)


(e)


(f)


(g)


(h)


4. b, c, e

EXERCISE 5 PAGE 361

1. (a) {9} (b) {6} (c) {9} (d) {0, 7}
(e) {8} (f) {3} (g) {3, 4, 5, 6 . . .} (h) {0, 1, 2}
(i) {5} (j) {0, 1, 2, 3 . . .}

2. (a) {12} (b) {2} (c) {9} (d) {6} (e) {10} (f) {3}
(g) {5} (h) {6} (i) {8} (j) {1}

EXERCISE 6

1. 3×▨+7=43; {12}
2. 8×▨−9=47; {7}
3. 25−2×▨=5; {10}
4. ▨×▨=70−6; {8}
5. 3×▨<29; {0, 1, 2 . . . 7, 8, 9}

EXERCISE 7

1. {(10, 0), (9, 1), (8, 2), . . . (1, 9), (0, 10)}
2. { }
3. {(0, 11), (1, 10), (2, 9), . . . (10, 1), (11, 0)}
4. {(0, 0), (3, 4), (6, 8), . . .}

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5. {(0, 24), (1, 21), (2, 18), . . . (7, 3), (8, 0)}
6. {(15, 0), (8, 1)}
7. {(0, 1), (1, 0)}
8. {(0, 18), (1, 17), (2, 16), . . . (17, 1), (18, 0)}
9. {(5, 0), (3, 3), (1, 6)}
10. {(11, 0), (9, 1), (7, 2), (5, 3), (3, 4), (1, 5)}

EXERCISE 8

1. {(0, 0), (3, 1), (6, 2), . . .}
2. {(10, 0), (12, 1), (14, 2), . . .}
3. ▨={0, 1, 2, 3 . . .} ▲={0, 1, 2, 3 . . .} (Any pair of whole numbers)
4. {(4, 0), (3, 1), (3, 0), (2, 2), (2, 1), (2, 0), (1, 3), (1, 2), (1, 1), (1, 0), (0, 4), (0, 3), (0, 2), (0, 1), (0, 0)}
5. ▨={3, 4, 5, 6, . . .} ▲={2, 3, 4, 5, . . .}
6. {(6, 0), (5, 0), (5, 1), (5, 2), (4, 0), (4, 1), (4, 2), (4, 3), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5) . . . (0, 8), (0, 9)}
7. {(36, 1), (18, 2), (12, 3), (9, 4), (6, 6), (4, 9), (3, 12), (2, 18), (1, 36)}
8. {(2, 1), (5, 3), (8, 5), (11, 7) . . .}
9. {(24, 1), (12, 2), (8, 3), (6, 4), (4, 6), (3, 8), (2, 12), (1, 24)}

10. {(0, 3), (1, 4), (2, 5), (3, 6) . . .}   11. {(0, 0), (5, 3), (10, 6), (15, 9), . . .}
12. {(0, 0), (1, 6), (2, 12), (3, 18) . . .}
13. {(0, 11), (0, 12), (0, 13) . . .
(1, 10), (1, 11), (1, 12) . . .
(2, 9), (2, 10), (2, 11) . . .
14 {(0, 0), (5, 2), (10, 4), (15, 6) . . .}

EXERCISE 9 PAGE 362

1. (a) { }   (b) {5, 6, 7, 8, 9, 10, 11, 12, 13}
(c) {2, 3, 4 . . .}   (d) {0, 1, 2, 3, 4 . . .}   (e) {0, 1, 2}
2. (a) {12}   (b) {0, 1, 2, 3, . . .}   (c) {27}   (d) {8}
(e) {11}   (f) {(3, 0), (4, 1), (5, 2), . . .}
(g) {10}   (h) {(0, 0), (1, 1), (2, 2), . . .}
(i) (0, 1, 2}   (j) {3}
3. (a) {1}   (b) {4}   (c) {9}   (d) {0}
(e) {0, 1, 2, 3, 4, 5}   (f) {0, 1, 2, 3, 4, 5, 6}   (g) {7, 8, 9, 10, . . .}
(h) {8, 9, 10, 11, . . .}   (i) {1, 2, 3, 4, 5, 6}
(j) {5}   (k) {1}   (l) {1}   (m) {1}
(n) {0, 1}   (o) {7, 8, 9, 10, . . .}   (p) {0, 1, 2, 3, . . .}
(q) { }   (r) {1, 3}   (s) {7, 8, 9, 10, . . .}
(t) {0, 1, 2, 3, 4, . . .}
4. (a) {4}   (b) {1}   (c) {3}   (d) {9}   (e) { }   (f) {15}   (g) {9}
(h) {13}   (i) {6}   (j) {0, 1, 2, 3}

EXERCISE 9 PAGE 363

5. (a) {(0, 0), (0, 1), . . . (0, 9), (1, 0), (1, 1), (1, 2), . . . (1, 8), (2, 0), (2, 1), (2, 2), . . . (2, 7), (3, 0), (3, 1), . . . (3, 6), (4, 0), (4, 1), . . . (4, 5), (5, 0), (5, 1), (5, 2), . . . (5, 4), (6, 0), (6, 1), . . . (6, 3), (7, 0), (7, 1), (7, 2), (8, 0), (8, 1), (9, 0)}
(b) {(0, 0), (1, 2), (2, 4), (3, 6), . . .}
(c) {(0, 6), (2, 5), (4, 4), (6, 3), (8, 2), (10, 1), (12, 0)}
(d) {(1, 1), (2, 2), (3, 3), . . .}
(e) {(1, 15), (3, 5), (5, 3), (15, 1)}
(f) {(1, 0), (2, 0), (3, 0), . . .
(2, 2), (3, 2), (4, 2), . . .
(3, 4), (4, 4), (5, 4), . . .
(g) {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0),
(2, 1), ...}
(h) {(0, 20), (1, 19), (2, 16), (3, 11), (4, 4)}
(i) {(0, 0), (1, 4), (2, 8), (3, 12), . . .}
(j) {(4, 0), (5, 1), (6, 2), (7, 3), . . .}
6. (a) {(10, 0), (9, 1), (8, 2), . . . (1, 9), (0, 10)}
(b) {(2, 1), (3, 3), (4, 5), (5, 7), . . .}
(c) {(0, 0), (0, 1), (0, 2), (0, 3), . . .
(1, 1), (1, 2), (1, 3), (1, 4), . . .
(2, 2), (2, 3), (2, 4), (2, 5), . . .
(3, 3), . . .}
(d) {(2, 0), (3, 1), (4, 2), (5, 3), . . .}
(e) {(0, 0), (0, 1), (0, 2), (0, 3), . . .}   (f) {(1, 0), (2, 2), (3, 4), (4, 6), . . .}

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(g) {(2, 0), (3, 0), (4, 0), (5, 0), (5, 1), (6, 0), (6, 1), (7, 0), (7, 1), (8, 0), (8, 1), (8, 2), . . .}

(h) {(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), . . .}
(i) {(0, 1), (1, 2), (2, 3), (3, 4), . . .}   (j) {(0, 0), (1, 1), (2, 2), (3, 3), . . .}
(k) {(11, 0), (0, 11), (11, 1), (1, 11), (11, 2), (2, 11), . . .
(10, 1), (1, 10), (10, 2), (2, 10), . . . (9, 2), (2, 9), (9, 3), (3, 9), . . .}
(l) {(3, 2)}
(m) $x=\{0, 1, 2, 3, 4, \ldots\}$ $y=\{0, 1, 2, 3, 4, \ldots\}$
(n) $x=\{0, 1, 2, 3, \ldots\}$ $y=\{0, 1, 2, 3, 4, \ldots\}$
(o) $x=\{0, 1, 2, 3, \ldots\}$ $y=\{0, 1, 2, 3, \ldots\}$
(p) {(24, 1), (12, 2), (8, 3), (6, 4), (4, 6), (3, 8), (2, 12), (1, 24)}
(q) {(0, 0), (0, 1), (0, 2), (0, 3), . . . (1, 0), (1, 1), (2, 0), (3, 0), (4, 0), . . .}
(r) {(6, 0), (7, 1), (8, 2), (9, 3), . . .}

EXERCISE 9 PAGE 364

(s) {(0, 0), (0, 1), (0, 2), (0, 3), . . . (1, 2), (1, 3), (1, 4), . . .
(2, 3), (2, 4), (2, 5), . . .
(t) {(0, 0), (3, 5), (6, 10), (9, 15), . . .}

EXERCISE 10 PAGE 364

1. (a) 36—commutative   (b) 15—associative
(c) 4—distributive   (d) 87—commutative
(e) 7—distributive

EXERCISE 11 PAGE 364

1. (a) {1, 7, 23}
(b) 49=7×7; 168=2×2×2×3×7; 1000=2×2×2×5×5×5;
156=2×2×3×13; 128=2×2×2×2×2×2×2
(c) {53, 59, 61, 67}
(d) 48—{1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
144—{1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144}
100—{1, 2, 4, 5, 10, 20, 25, 50, 100}
78—{1, 2, 3, 6, 13, 26, 39, 78}
198—{1, 2, 3, 6, 9, 11, 18, 22, 33, 66, 99, 198}
(e) 3

EXERCISE 11 PAGE 365

2. (a) 7   (b) 5   (c) 17   (d) 7   (e) 13   (f) 17
3. (a) 12   (b) 55   (d) 231   (f) 44   (h) 132   (i) 770   (j) 660
Factors of each are also factors of 4620
4. (a) 28   (b) 38   (c) 55   (d) 220   (e) 133   (f) 75
(g) 130   (h) 30
5. (a) 30   (b) 50   (c) 18   (d) 24   (e) 19   (f) 66
(g) 56   (h) 48   (i) 44   (j) 60   (k) 25   (l) 15
6. 28

EXERCISE 12 PAGE 365

1. (a) 18   (b) 6   (c) 12   (d) 21   (e) 36   (f) 42
(g) 2   (h) 15   (i) 22
2. (a) 12   (b) 39   (c) 14   (d) 25   (e) 100   (f) 106
(g) 63   (h) 18   (i) 15

EXERCISE 12 PAGE 366

3. (a) 12 (b) 13 (c) 17 (d) 30 (e) 48 (f) 11 (g) 3 (h) 19
(i) 28 (j) 63 (k) 1 (l) 25 (m) 36 (n) 48 (o) 32 (p) 27
(q) 98 (r) 12 (s) 1 (t) 144

4. (a) 36 (b) 75 (c) 96 (d) 84 (e) 60 (f) 300
(g) 144 (h) 360 (i) 210

5. (a) 6 (b) 24 (c) 60 (d) 24 (e) 30
(f) 216 (g) 72 (h) 33 (i) 20 (j) 90
(k) 45 (l) 50 (m) 16 (n) 35 (o) 42
(p) 126 (q) 240 (r) 65 (s) 36 (t) 72

EXERCISE 13–PAGE 366

1. (a) $\frac{2}{10}=\frac{3}{15}=\frac{5}{25}$ (b) $\frac{15}{27}=\frac{30}{54}=\frac{60}{108}$ (c) $\frac{16}{16}=\frac{64}{64}=\frac{256}{256}$
(d) $\frac{14}{24}=\frac{21}{36}=\frac{28}{48}$ (e) $\frac{18}{22}=\frac{27}{33}=\frac{36}{44}$ (f) $\frac{8}{2}=\frac{12}{3}=\frac{16}{4}$
(g) $\frac{10}{16}=\frac{15}{24}=\frac{20}{32}$ (h) $\frac{10}{12}=\frac{15}{18}=\frac{20}{24}$ (i) $\frac{6}{14}=\frac{9}{21}=\frac{12}{28}$
(j) $\frac{22}{66}=\frac{13}{39}=\frac{44}{132}$ (k) $\frac{10}{18}=\frac{15}{27}=\frac{20}{36}$ (l) $\frac{22}{42}=\frac{33}{63}=\frac{44}{84}$
(m) $\frac{6}{26}=\frac{9}{39}=\frac{12}{52}$ (n) $\frac{14}{30}=\frac{21}{45}=\frac{28}{60}$

EXERCISE 13–PAGE 367

2. (a) 28 (b) 2 (c) 20 (d) 57 (e) 11
(f) 18 (g) 253 (h) 1 (i) 15 (j) 20
(k) 190 (l) 132 (m) 408 (n) 25 (o) 1

3. (a) $\frac{6}{16}$ (b) $\frac{45}{25}$ (c) $\frac{10}{55}$ (d) $\frac{24}{2}$ (e) $\frac{65}{15}$
(f) $\frac{0}{4}$ (g) $\frac{32}{34}$ (h) $\frac{12}{6}$ (i) $\frac{3}{18}$ (j) $\frac{48}{8}$

4. (a) $\frac{1}{2}$ (b) 3 (c) $\frac{7}{8}$ (d) $\frac{11}{12}$ (e) $\frac{7}{18}$
(f) $\frac{2}{3}$ (g) $\frac{17}{20}$ (h) $\frac{23}{25}$ (i) $\frac{1}{2}$ (j) $\frac{5}{6}$
(k) $\frac{4}{5}$ (l) $\frac{27}{76}$ (m) $\frac{5}{6}$ (n) $\frac{11}{17}$ (o) $\frac{27}{28}$
(p) $\frac{7}{9}$ (q) $\frac{4}{5}$ (r) $\frac{2}{3}$ (s) $\frac{9}{10}$ (t) $\frac{1}{10}$

5. (a) $\frac{2}{15}, \frac{4}{15}, \frac{7}{10}$ (b) $\frac{4}{5}, \frac{13}{15}, \frac{9}{10}$ (c) $\frac{1}{4}, \frac{2}{3}, \frac{4}{5}$
(d) $\frac{2}{3}, \frac{7}{8}, \frac{11}{12}$ (e) $\frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (f) $\frac{2}{5}, \frac{5}{6}, \frac{7}{8}$
(g) $\frac{3}{80}, \frac{3}{16}, \frac{3}{8}$ (h) $\frac{5}{12}, \frac{11}{18}, \frac{5}{8}$ (i) $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$
(j) $\frac{4}{9}, \frac{5}{8}, \frac{2}{3}$ (k) $\frac{11}{24}, \frac{9}{15}, \frac{7}{10}$ (l) $\frac{1}{5}, \frac{2}{7}, \frac{1}{2}$

EXERCISE 14–PAGE 367

1. (a) $2\frac{4}{5}$ (b) $2\frac{3}{4}$ (c) $2\frac{1}{3}$ (d) 12 (e) 3 (f) $3\frac{2}{3}$
(g) $12\frac{1}{3}$ (h) $3\frac{3}{7}$ (i) 2 (j) 3 (k) 3 (l) $3\frac{1}{30}$

2. (a) $1\frac{7}{40}$ (b) $1\frac{4}{9}$ (c) $1\frac{4}{9}$ (d) $1\frac{23}{72}$ (e) $1\frac{23}{30}$ (f) $5\frac{3}{10}$

EXERCISE 14–PAGE 368

(g) $1\frac{35}{132}$ (h) $6\frac{19}{24}$ (i) $5\frac{5}{8}$ (j) $1\frac{32}{45}$ (k) $1\frac{17}{120}$ (l) $1\frac{41}{44}$
(m) $1\frac{59}{60}$ (n) 2 (o) 5 (p) $2\frac{44}{63}$ (q) $5\frac{11}{15}$ (r) $2\frac{36}{55}$

3. (a) $1\frac{3}{8}$ (b) $2\frac{1}{4}$ (c) $1\frac{41}{70}$ (d) $\frac{47}{60}$ (c) $2\frac{1}{120}$ (f) $1\frac{109}{120}$
(g) $1\frac{133}{152}$ (h) $1\frac{25}{63}$ (i) $1\frac{19}{72}$

4. (a) $7\frac{4}{15}$ (b) $11\frac{41}{120}$ (c) $5\frac{29}{30}$ (d) $14\frac{5}{18}$ (e) $5\frac{1}{3}$ (f) $8\frac{4}{9}$
(g) $11\frac{5}{8}$ (h) $5\frac{31}{60}$ (i) $7\frac{43}{45}$ (j) $13\frac{47}{72}$

5. (a) $1\frac{11}{12}$ (b) $1\frac{19}{24}$ (c) $\frac{3}{4}$ (d) $2\frac{1}{24}$ (e) $2\frac{17}{72}$ (f) $7\frac{1}{20}$
(g) $10\frac{5}{8}$ (h) $7\frac{11}{30}$ (i) $11\frac{33}{40}$ (j) $17\frac{271}{315}$ (k) $17\frac{28}{195}$ (l) $10\frac{11}{180}$
(m) $5\frac{43}{130}$ (n) $7\frac{13}{60}$ (o) $9\frac{47}{72}$ (p) $9\frac{13}{126}$

6. $175\frac{7}{16}$

EXERCISE 15–PAGE 368

1. (a) 3 (b) $4\frac{7}{8}$ (c) $5\frac{1}{5}$ (d) $7\frac{13}{24}$ (e) $\frac{5}{12}$ (f) $\frac{59}{60}$
(g) $\frac{43}{72}$ (h) $4\frac{5}{39}$ (i) $1\frac{17}{72}$ (j) $1\frac{46}{63}$ (k) $\frac{49}{72}$ (l) $\frac{1}{10}$
(m) $1\frac{23}{45}$ (n) $8\frac{9}{10}$ (o) $4\frac{23}{44}$ (p) $1\frac{1}{6}$ (q) $\frac{139}{144}$ (r) $7\frac{191}{240}$

EXERCISE 15–PAGE 369

2. (b) $\frac{2}{7}$ (c) $\frac{7}{9}$ (d) $\frac{7}{13}$ (e) $\frac{13}{15}$ (f) $\frac{1}{4}$ (g) 1
(h) $1\frac{2}{5}$ (i) 1 (j) $1\frac{2}{5}$
3. (b) $\frac{7}{12}$ (c) $\frac{47}{72}$ (d) $\frac{1}{10}$ (e) $\frac{65}{168}$ (f) $\frac{11}{24}$ (g) $\frac{3}{10}$
(h) $2\frac{1}{8}$ (i) $1\frac{3}{13}$ (j) $\frac{7}{24}$ (k) $2\frac{1}{6}$ (l) $1\frac{1}{35}$ (m) $\frac{53}{56}$
4. (a) $\frac{9}{10}$ (b) $\frac{89}{90}$ (c) $4\frac{3}{7}$ (d) $1\frac{29}{30}$ (e) $\frac{1}{8}$ (f) $1\frac{103}{120}$
(g) $\frac{11}{12}$ (h) $\frac{14}{45}$ (i) $1\frac{19}{36}$
5. (a) $\frac{7}{40}$ (b) $\frac{39}{40}$ (c) $4\frac{1}{36}$ (d) $2\frac{17}{20}$ (e) $\frac{13}{60}$ (f) $1\frac{40}{91}$
(g) $5\frac{7}{8}$ (h) $\frac{5}{19}$ (i) $\frac{26}{45}$
6. (a) $2\frac{3}{40}$ (b) $15\frac{14}{15}$

EXERCISE 15–PAGE 370

(c) $\frac{5}{18}$ (d) $12\frac{31}{120}$ (e) $1\frac{9}{28}$ (f) $\frac{7}{20}$

EXERCISE 16–PAGE 370

1. (a) $1\frac{1}{2}$ (b) 14 (c) $35\frac{1}{2}$ (d) $10\frac{2}{5}$ (e) 5 (f) $\frac{5}{9}$
(g) $\frac{1}{3}$ (h) 9 (i) $\frac{9}{32}$ (j) $2\frac{3}{4}$ (k) $3\frac{7}{16}$ (l) $1\frac{3}{5}$
(m) 3 (n) 6 (o) 1 (p) $4\frac{1}{2}$ (q) 27 (r) $8\frac{1}{4}$
(s) $1\frac{5}{16}$ (t) $34\frac{2}{27}$ (u) $54\frac{69}{80}$
2. (a) $\frac{8}{15}$ (b) $\frac{3}{4}$ (c) $1\frac{1}{2}$ (d) $5\frac{1}{3}$ (e) $25\frac{1}{3}$ (f) 55
(g) 252 (h) 23 (i) $103\frac{1}{5}$ (j) $8\frac{2}{3}$ (k) 13 (l) $\frac{1}{3}$
(m) $8\frac{31}{160}$ (n) $12\frac{1}{3}$ (o) 1
3. (a) $2\frac{2}{3}$ (b) 7 (c) $1\frac{1}{5}$ (d) 11 (e) $5\frac{1}{2}$ (f) $10\frac{1}{2}$
(g) 15 (h) $25\frac{5}{9}$ (i) 11 (j) $3\frac{2}{15}$
4. 33
5. (a) $3\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) $1\frac{1}{4}$ (e) $2\frac{3}{4}$ (f) $\frac{1}{2}$
(g) $\frac{5}{8}$ (h) 8 (i) $5\frac{3}{4}$ (j) $4\frac{1}{2}$

EXERCISE 17–PAGE 371

1. (a) 4 (b) 20 (c) $3\frac{15}{23}$ (d) 2 (e) $1\frac{1}{14}$ (f) $1\frac{1}{14}$
(g) $\frac{1}{40}$ (h) $\frac{1}{9}$ (i) $\frac{1}{4}$ (j) $\frac{3}{8}$ (k) $\frac{1}{2}$ (l) 24
(m) $17\frac{1}{7}$ (n) $5\frac{1}{2}$ (o) $\frac{3}{10}$ (p) $\frac{2}{15}$ (q) $\frac{11}{12}$ (r) $\frac{4}{9}$
(s) $3\frac{1}{5}$ (t) $11\frac{4}{21}$
2. (a) 2 (b) $1\frac{1}{2}$ (c) $13\frac{1}{2}$ (d) \$14.40 (e) $\frac{1}{48}$ (f) $\frac{1}{100}$
(g) $\frac{1}{2}$ (h) $\frac{1}{3}$ (i) 2 (j) $\frac{1}{2}$ (k) $4\frac{1}{2}$ (l) $1\frac{1}{7}$
(m) $1\frac{1}{5}$ (n) $\frac{8}{9}$ (o) $1\frac{1}{5}$ (p) $\frac{2}{25}$ (q) $\frac{1}{16}$ (r) $\frac{5}{6}$
(s) $\frac{5}{6}$ (t) $\frac{1}{9}$
3. (a) $>$ (b) $<$ (c) $=$ (d) $>$ (e) $>$ (f) $<$
(g) $<$ (h) $=$
4. (a) $\frac{5}{16}, \frac{3}{8}, \frac{7}{16}$ (b) $\frac{17}{72}, \frac{1}{4}, \frac{19}{72}$ (c) $1\frac{5}{12}, 1\frac{1}{2}, 1\frac{2}{3}$ (d) $\frac{3}{4}, \frac{15}{16}, 1\frac{1}{8}$
(e) $1\frac{13}{24}, 1\frac{7}{12}, 1\frac{15}{24}$ (f) $\frac{41}{72}, \frac{43}{72}, \frac{7}{12}$ (g) $\frac{37}{40}, \frac{19}{20}, \frac{39}{40}$ (h) $\frac{1}{3}, \frac{1}{2}, \frac{13}{24}$ etc.
5. 30 6. \$9.46

EXERCISE 18–PAGE 371

1. (a) ones (b) tenths (c) hundreds
(d) hundredths (e) thousandths (f) hundred-thousandths
(g) ten-thousandths (h) ten-thousandths (i) millions
(j) ones

EXERCISE 18–PAGE 372

2. (a) $(5\times 1)+(2\times \frac{1}{10})+(8\times \frac{1}{100})=5\frac{28}{100}$ (b) $(1\times 10)+(4\times 1)+(3\times \frac{1}{10})=14\frac{3}{10}$
(c) $(2\times 1)+(0\times \frac{1}{10})+(6\times \frac{1}{100})=2\frac{6}{100}$
(d) $(1\times 10)+(3\times 1)+(9\times \frac{1}{10})+5\times \frac{1}{100}=13\frac{95}{100}$
(e) $(2\times 1)+(0\times \frac{1}{10})+(4\times \frac{1}{100})+(7\times \frac{1}{1000})=2\frac{47}{1000}$
(f) $(1\times 10)+(6\times 1)+(0\times \frac{1}{10})+(0\times \frac{1}{100})+(8\times \frac{1}{1000})=16\frac{8}{1000}$
(g) $(7\times 1)+(2\times \frac{1}{10})+(9\times \frac{1}{100})+(6\times \frac{1}{1000})=7\frac{296}{1000}$
(h) $(1\times 10)+(5\times 1)+(8+\frac{1}{10})+(2\times \frac{1}{100})+(4\times \frac{1}{1000})=15\frac{824}{1000}$
(i) $(3\times 1)+(0\times \frac{1}{10})+(0\times \frac{1}{100})+(1\times \frac{1}{1000})+(0\times \frac{1}{10000})+(1\times \frac{1}{100000})=3\frac{101}{100000}$
(j) $(1\times 10)+(7\times 1)+(5\times \frac{1}{10})+(6\times \frac{1}{100})+(7\times \frac{1}{1000})+(2\times \frac{1}{10000})=17\frac{5672}{10000}$
(k) $(9\times 1)+(8\times \frac{1}{10})+(8\times \frac{1}{100})+(7\times \frac{1}{1000})=9\frac{887}{1000}$
(l) $(2\times 10)+(8\times 1)+(8\times \frac{1}{10})+(4\times \frac{1}{100})+(3\times \frac{1}{1000})=28\frac{843}{1000}$
(m) $(8\times 1)+(9\times \frac{1}{10})+(8\times \frac{1}{100})+(7\times \frac{1}{1000})=8\frac{987}{1000}$
(n) $(3\times 10)+(5\times 1)+(9\times \frac{1}{10})+(0\times \frac{1}{100})+(1\times \frac{1}{1000})+(4\times \frac{1}{10000})=35\frac{9014}{10000}$
(o) $(1\times 10)+(1\times 1)+(0\times \frac{1}{10})+(0\times \frac{1}{100})+(0\times \frac{1}{1000})+(1\times \frac{1}{10000})+9\times \frac{1}{100000})$
$=11\frac{19}{100000}$
(p) $(4\times 10)+(1\times 1)+(0\times \frac{1}{10})+(8\times \frac{1}{100})+(0\times \frac{1}{1000})+(8\times \frac{1}{10000})=41\frac{808}{10000}$

3. (a) $\frac{47}{10}$ (b) $\frac{59}{10}$ (c) $\frac{113}{10}$ (d) $\frac{162}{10}$ (e) $\frac{704}{100}$ (f) $\frac{932}{100}$
(g) $\frac{1580}{100}$ (h) $\frac{209}{100}$ (i) $\frac{69}{1000}$ (j) $\frac{3947}{1000}$ (k) $\frac{2909}{1000}$ (l) $\frac{8354}{1000}$

4. (a) .3 (b) .29 (c) .47 (d) .453 (e) 34.14 (f) 900.099
(g) .1068 (h) 54.0082

5. (a) .34 (b) .725 (c) .8125 (d) .72 (e) .515625
(f) .171875 (g) .024 (h) .595

EXERCISE 19–PAGE 372

1. (a) 1.0 (b) 6.29 (c) 9.35 (d) 23.09 (e) 7.619 (f) 56.6896
(g) 8.2112 (h) .1249 (i) 17.8 (j) 34.41 (k) 62.965 (l) 44.8840

2. (a) 4.5 (b) 6.7 (c) 8.7 (d) 2.6

EXERCISE 19–PAGE 373

(e) .4 (f) 7.64 (g) 33.19 (h) 8.92 (i) 8.880 (j) 44.294
(k) 4.984 (l) 8.489 (m) 3.4462 (n) .1615 (o) .2818 (p) 4.3505

3. (a) 335.62 (b) 43.9584 (c) 10.50576 (d) .0019734
(e) 1.549822 (f) .012321 (g) 5.84658 (h) 377.09496
(i) 24.0486 (j) 3.5118936 (k) .623562 (l) 144.93828

4. (a) 2.03 (b) .4 (c) 4.82 (d) .536 (e) 87.6 (f) 109
(g) .58 (h) .0924 (i) 4.768 (j) 2.34 (k) .1

EXERCISE 20–PAGE 373

1. (a) 1189.509 (b) 53.1463 (c) 2627.59 (d) 184.3203
(e) 23.25957 (f) 834.59531 (g) 297.81081

2. (a) .09057 (b) 82.9894 (c) 1.081 (d) .001
(e) 6.6034 (f) 509.906 (g) 9.981 (h) 21.8864
(i) 1.015 (j) .095 (k) 1.967

EXERCISE 20–PAGE 374

3. (a) .061305 (b) 8.216 (c) 1.64829 (d) 2.20242
(e) 4.824 (f) 325.9032 (g) 8740.08 (h) 260.833
(i) 2.051973 (j) 346.0695 (k) 21662.41 (l) .00324258
(m) .1221144 (n) .837644

4. (a) 10.944 (b) 10.944 (c) .10944 (d) 109.44
(e) 10.944 (f) .010944 (g) 10.944 (h) 10.944
(i) .0010944 (j) 1094400 (k) 109440 (l) .0010944

5. (a) 1.582 (b) 22.6 (c) $2.6\dot{1}\dot{8}$ (d) 42.876
(e) $101.\dot{5}4\dot{0}$ (f) 15.868 (g) .356 (h) 36.947
(i) 2.122 (j) .191 (k) 3.308 (l) 325.31
(m) 4.965 (n) 31.867 (o) 32.282 (p) .62
(q) 5.789 (r) 11.551

EXERCISE 21–PAGE 374

1. $.\dot{2}$; .085; $.\dot{8}5714\dot{2}$; .972; $.\dot{2}380\dot{9}$; $.91\dot{6}$; .515625; $.31\dot{6}$; .69375; $.4\dot{6}$; $.\dot{7}\dot{2}$; .5975; .9375; $.9\dot{6}$

2. 39 ft. 3. .023 in. 4. 82.95 mi. 5. 2.79

EXERCISE 22–PAGE 375

1. 14.756 2. $307.40 3. $5.50 4. 1974.13 lb. 5. 24.4408

EXERCISE 23–PAGE 375

1. $4\frac{3}{4}$ lb. 2. $154.00 3. $\frac{3}{16}$ in. 4. 3 5. $8.40

EXERCISE 24–PAGE 375

1. (i) $\frac{3}{50}$; (3:50) (ii) $\frac{88}{1}$; (88:1) (iii) $\frac{15}{4}$; (15:4)
(iv) $\frac{69}{5}$; (69:5) (v) $\frac{1}{700}$; (1:700)

EXERCISE 24–PAGE 376

2. (a); (b); (e); (h); (i); (k); (l)

3. (a) 1 (b) 5 (c) 8 (d) 39 (e) $1\frac{1}{2}$ (f) $1\frac{1}{4}$
(g) 1 (h) $2\frac{1}{4}$ (i) 1 (j) 2 (k) 5 (l) 6

4. $17\frac{1}{2}$ hr. 5. 2

1. (a) 12 (b) 36 (c) 12 (d) 7 (e) 39 (f) 10 (g) 24 (h) $7\frac{1}{2}$
(i) 25 (j) 14 (k) 6 (l) 9 (m) 22 (n) 25 (o) 16 (p) 27
(q) 1 (r) 4 (s) 5 (t) 24 (u) 12 (v) 16 (w) 30 (x) 6

EXERCISE 25–PAGE 377

(y) 20 (z) 51 (aa) 5 (bb) 49 (cc) 30 (dd) 2

EXERCISE 25–PAGE 377

2. $\frac{1}{3}$, (1, 3), 1:3 $\frac{3}{1}$, (3, 1), 3:1
$\frac{1}{16}$, (1, 16), 1:16 $\frac{16}{1}$, (16, 1), 16:1
$\frac{3}{10}$, (3, 16), 3:16 $\frac{16}{3}$, (16, 3), 16:3
$\frac{1}{5}$, (1, 5), 1:5 $\frac{5}{1}$, (5, 1), 5:1
$\frac{3}{8}$, (3, 8), 3:8 $\frac{8}{3}$, (8, 3), 8:3
$\frac{1}{4}$, (1, 4), 1:4 $\frac{4}{1}$, (4, 1), 4:1
$\frac{1}{5}$, (1, 5), 1:5 $\frac{5}{1}$, (5, 1), 5:1
$\frac{1}{8}$, (1, 8), 1:8 $\frac{8}{1}$, (8, 1), 8:1
$\frac{1}{1}$, (1, 1), 1:1 $\frac{1}{1}$, (1, 1), 1:1
$\frac{1}{16}$, (1, 16), 1:16 $\frac{16}{1}$, (16, 1), 16:1

3. \$6100.00 4. $1\frac{1}{4}$ pt. 5. 199.52 m.p.h. 6. 6. oz. 7. 9:1

EXERCISE 26–PAGE 378

1. 33%; 19%; 206%; 1000%; $1\frac{3}{4}$%; $\frac{7}{8}$%; 34%; 90%; 25%; 60%; 10%; 2%; 122%; 225%; 180%; 370%; 655%; 350%

2. $\frac{6}{25}$; $\frac{1}{4}$; $\frac{9}{50}$; $\frac{11}{25}$; $\frac{3}{4}$; $\frac{16}{25}$; $1\frac{3}{20}$; 3; $1\frac{1}{10}$; $5\frac{1}{2}$; 2; $2\frac{4}{25}$

3. 7%; 14%; 96%; 50%; 75%; 80%; 120%; 358%; 792%; 560%; 284%; 321%; .4%; 5.8%; 72.5%; 105.5%; 321.8%; 564.4%

4. .15; .75; .60; .78; .95; .02; .136; .0485; .032; .05615; 2.09; 1.00

5. $66\frac{2}{3}$%; 60%; $83\frac{1}{3}$%; $87\frac{1}{2}$%; $91\frac{2}{3}$%; $56\frac{1}{4}$%; 180%; $337\frac{1}{2}$%; 455%; $566\frac{2}{3}$%; $473\frac{1}{3}$%; $157\frac{1}{7}$%; 240%; $162\frac{1}{2}$%; $113\frac{1}{3}$%; $335\frac{5}{7}$%; 116%; $612\frac{1}{2}$%

EXERCISE 27–PAGE 378

1. (a) 19.5 (b) 15 (c) 26 (d) 225 (e) 52 (f) 456
(g) 30 (h) 110.7 (i) $1\frac{3}{5}$ (j) 1163.4 (k) $184\frac{1}{2}$ (l) 90

2. (a) 90 (b) 175 (c) 75 (d) 133.3 (e) 75 (f) 166.7
(g) 8.3 (h) 66.7 (i) 87.5 (j) 200 (k) 112.5 (l) 52.9

EXERCISE 27–PAGE 379

3. (a) 240 (b) 168 (c) 448 (d) 140 (e) 60 (f) 56
(g) 20 (h) 864 (i) 237.5 (j) 60 (k) 220 (l) 400

4. (a) 110 (b) $66\frac{2}{3}$ (c) 105 (d) 35 (e) 200 (f) $181\frac{9}{11}$
(g) 100 (h) 125 (i) 22.68 (j) 172 (k) 1.62 (l) $1\frac{2}{5}$

5. (a) $83\frac{1}{3}$ (b) 1.5 (c) 3 (d) 3 (e) 3 (f) 20
(g) 100 (h) 100 (i) $16\frac{2}{3}$ (j) .88 (k) 126 (l) $4\frac{1}{2}$

EXERCISE 28–PAGE 379

1. $37\frac{1}{2}$ 2. 200 3. 90 4. 3,000 27,000 5. 14

EXERCISE 29–PAGE 379

1. 672 2. \$318.40 3. \$100.80 4. 20 5. \$15,000.00

EXERCISE 30–PAGE 380

1. \$2.40 2. \$270.00 3. \$180.00 4. $16\frac{2}{3}$ 5. \$28.33

EXERCISE 31–PAGE 380

1. \$2.19   2. \$14.85

EXERCISE 31–PAGE 381

3. 4   4. \$59.44   5. 16

EXERCISE 32–PAGE 381

2. (a) 7.7″   (b) 10′ 6″   (c) 51   (d) 60′

EXERCISE 32–PAGE 382

3. (a) right   (b) straight
4. 35°, 10°, 75°
5. (a) 90°—right   (b) 45°—acute   (c) 135°—obtuse
(d) 45° acute   (e) 180°—straight   (f) 30°—acute

EXERCISE 33–PAGE 382

1. acute   3. equal angles

EXERCISE 34–PAGE 383

1. $112\frac{1}{2}''$ 2. 30′   3. D = 5″   C = 15.708″   4. $73\frac{1}{3}$ ft.   5. 1.0″

EXERCISE 35–PAGE 384

2. $1\frac{1}{2}$ sq. in.   3. 4. 61″   4. .75″   5. .75″

EXERCISE 36–PAGE 384

1. \$382.86   2. 240   3. $9\frac{1}{3}$ yd.   4. 113 sq. ft.   5. $177\frac{7}{9}$ sq. ft.

EXERCISE 37–PAGE 384

1. 7 sq. in.

EXERCISE 37–PAGE 385

2. $19\frac{1}{2}$ sq. in.; $\frac{13}{96}$ sq. ft.; $\frac{13}{864}$ sq. yd.
1620 sq. in.; $11\frac{1}{4}$ sq. ft.; $1\frac{1}{4}$ sq. yd.
90 sq. in.; $\frac{5}{8}$ sq. ft.; $\frac{5}{72}$ sq. yd.
3. 1.1 acres   4. \$66.00   5. $3\frac{3}{4}$ sq. in.

EXERCISE 38–PAGE 385

1. (a) 360 cu. ft.   (b) $13\frac{1}{3}$ cu. yd.   2. 3375

EXERCISE 38–PAGE 386

3. \$72.92   4. \$1260.00
5. A — $14\frac{7}{32}$ cu. in.;   B — 120 cu. ft.;   C — 2 yd.;   D — 8″
E — 1 ft.   F — $10\frac{5}{12}$ ft.;   G — $1\frac{2}{3}$ cu. yd.;   H — $2\frac{2}{3}''$;
I — $2\frac{2}{3}$ yd.;   J — 7.65625 ft.

EXERCISE 39–PAGE 386

1. (a) 67 (b) 96 (c) 126 (d) 16 (e) 21 (f) 22 (g) 33 (h) 4
(i) 30 (j) 10 (k) 20 (l) 38 (m) 51 (n) 0 (o) 29 (p) 26
(q) 3 (r) 45 (s) 5 (t) 33

EXERCISE 39–PAGE 387

2. (a) 72 (b) 2 (c) 14 (d) 32 (e) 2 (f) 32 (g) 2 (h) 120
(i) 375 (j) 19 (k) 25 (l) 1 (m) 8 (n) 5 (o) 1 (p) 2
(q) 56 (r) 1 (s) 0 (t) 110

3. (a) $11\frac{1}{2}$ (b) 72 (c) 639 (d) 2 (e) 499 (f) 84
(g) 356 (h) 3,562,192 (i) 4 (j) 42

4. (a) 3 (b) 2 (c) 1 (d) 3 (e) 3 (f) 2
(g) 20 (h) $4\frac{1}{2}$ (i) 1 (j) $10\frac{2}{3}$ (k) 2 (l) 1
(m) 8 (n) 6 (o) 1 (p) 1 (q) 4 (r) 0

EXERCISE 40–PAGE 388

1. (a) 11 (b) 3 (c) 6 (d) 5 (e) 51 (f) 19 (g) 59 (h) 7
(i) 4 (j) 6 (k) 4 (l) 6 (m) 5 (n) 0 (o) 6 (p) 20
(q) 70 (r) 7 (s) 30 (t) 0

2. (a) 48 (b) 429 (c) 639 (d) 76 (e) 12

3. (a) 7 (b) 10 (c) 25 (d) 44 (e) 6 (f) 12 (g) 128 (h) 4
(i) 1 (j) 3

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